Advanced Micro I

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EXERCISE 1:

1. Consider the following potential properties of a binary relation R on a set X

(i) Completeness
(ii) Transitivity
Which of the above properties is satisfied by each of the following binary relations?
a) $X = \mathbb{R}^2_+$, xRy iff (iff = if and only if) $x \geq y$
b) $X = \mathbb{R}^n_+$, xRy iff $\min\{x_1, x_2, ..., x_n\} \geq \min\{y_1, y_2, ..., y_n\}$
c) $X = \mathbb{R}_+$, xRy iff $x > y$
d) $X = \mathbb{R}^2_+$, xRy for all $x, y \in X$
e) $X = \mathbb{R}_+$, xRy iff $\frac{x}{5} - \left[\frac{x}{5}\right] \geq \frac{y}{5} - \left[\frac{y}{5}\right]$, where $[z]$ (integer part of z) is the largest integer smaller than z.
f) $X = \mathbb{R}^2_+$, xRy iff either $x_1 > y_1$, or if $x_1 = y_1$ then $x_2 \geq y_2$
g) X - arbitrary, 2^X is the set of all subsets of X. For any $A, B \in 2^X$ ARB iff $A \subset B$

2. Consider the function $f : X \to Y$

$$f(x) = x^2$$

for the following X and Y determine whether it is injective, surjective or bijective.

a) $X = \mathbb{R}, Y = \mathbb{R}$
b) $X = \mathbb{R}_+, Y = \mathbb{R}_+$
c) $X = \mathbb{R}, Y = \mathbb{R}_+$
d) $X = \mathbb{R}_+, Y = \mathbb{R}$

3. Show that if a binary relation R is transitive, its symmetric and asymmetric parts, I and P are also transitive.

4. *(this is largely a reviw of your Eco 3 and is here to make sure you remember these functional forms well; please make sure you draw level curves)*
Consider the following functions $f : \mathbb{R}^2_+ \to \mathbb{R}_+$
a) $f(x) = x_1^\alpha x_2^\beta$
b) $f(x) = (\alpha x_1^\rho + \beta x_2^\rho)^{\frac{1}{\rho}}$
c) $f(x) = \min\{x_1, x_2\} + \alpha \max\{x_1, x_2\}$
For which values of the parameters α, β, ρ are these functions concave, convex, quasi-concave, quasi-convex? You may restrict attention to $\alpha, \beta \geq 0$.