EXERCISE 3:

1.Let $X = \mathbb{R}^2_+$ and let preferences be represented by the following utility function:

$$u(x_1, x_2) = \min\{x_1, x_2\} + \alpha \max\{x_1, x_2\}, \alpha \geq 0$$

Derive the Walrasian and Hicksian demand, indirect utility and expenditure functions.

2. Consider a demand function $x : \mathbb{R}^4_{++} \to \mathbb{R}^3_+$ given by

$$x_1(p, w) = x_2(p, w) = \frac{w}{2(p_1 + p_2)}$$
$$x_3(p, w) = \frac{w}{2p_3}$$

Can this be a demand of a rational consumer? If yes, calculate a corresponding expenditure and indirect utility functions. What can you say about the preferences of this consumer?

3 Consider the following function $x : \mathbb{R}^3_{++} \to \mathbb{R}^2$ (note, negative consumption is allowed)

$$x_1(p, w) = a\frac{w}{p_1} - \beta\frac{p_1}{p_2}$$
$$x_2(p, w) = \gamma\left(\frac{p_1}{p_2}\right)^\delta$$

where w is individual wealth and p_1, p_2 are prices. What restrictions on the parameters α and β are needed for this to be a possible Marshallian demand function of a rational consumer.

4. Two individuals, Anna and Boris, consume two goods, honey and milk and each has an income of \$1000. When the price vector is (10,10), then Anna consumes 25 units of honey and 75 units of milk, while Boris consumes 75 units of honey and 25 units of milk. When the price vector is (15,5), Anna's consumption bundle is (40,80), while Boris's is (64,8).
a) do individual demands obey WARP?
b) does the aggregate demand obey WARP?