This is the short summary of the minimum you need to know about preference relations and utility functions. If any of this is unclear, make sure you talk to me soon.

1 What you need to know about preferences

1. A **consumption space** X is the set of all alternatives that you might be choosing from. **Consumption bundle** $x \in X$ is a complete description of one possible alternative.

In this class we have consumption spaces that are

a) finite: $X = \{apple, orange, banana\}$

b) one-dimensional continuum: $X \subset \mathbb{R}$ (usual interpretation of a consumption bundle: quantity of a single good, such as the amount of money - *e.g., $3 pesos*).

c) multi-dimensional continuum: $X \subset \mathbb{R}^n$ (usual interpretation of a consumption bundle a vector of consumption of different goods, where n is the number of goods - *e.g.* $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ - *2 apples and 3 oranges*)

Note: if I want to talk about some particular consumption bundle (e.g. "3 apples, 4 oranges, and a cup of tea"), I would specify it exactly. When I write "for any $x, y \in X$" I mean for any combination of two bundles; I don't mean that there is a specific bundle called x and another one called y (I know, it may seem ridiculous to review this here, but a lot of you seem to need it).

2. Given a consumption space X every individual has a **preference relation** "$\succsim$" defined on it, so that for any two consumption bundles x and y:

$$x \succsim y$$

reads "x is at least as good as y". Note that "$\succ$" is the "strict" part ("$x \succ y$" reads "x is strictly better than y" and "$\sim$" is the indifference part.

3. Our basic hypothesis is that preferences are **rational. Rationality** is defined to mean that both of the following hold::

A. Complete : you can always compare any two consumption bundles (one will be better than another, or you will be indifferent) : For any x, y either $x \succsim y$ or $y \succsim x$ (or, perhaps, both - which means indifference).

B. Transitive : if you say that you like one bundle at least as much as you like another, and you like that bundle at least as much as you like some third bundle, than you have to like the first bundle at least as much as you like the third bundle: For any x, y, z , if $x \succsim y$ and $y \succsim z$ then it has to be that $x \succsim z$.

NOTE 1: if $X = \mathbb{R}^2_+$ (the case for which we draw most of the diagrams), completeness means that once you draw an indifference curve, everything must be either on it, better than it, or worse than it. Transitivity means that indifference curves don't intersect.

NOTE 2: in this class rationality **never** means anything else: monotonicity, local non-satiation, continuity, convexity, etc. are not part of rationality.

4. If the consumption space is $X \subset \mathbb{R}^n$, $n \geq 1$, then we studied other properties that preferences may (but don't have to) satisfy. These are: **strict monotonicity** (or, somewhat weaker, **local non-satiation**), **convexity**, **strict convexity** and **continuity**. For definitions you should consult the textbook.

2 Utility function representation

1. It is hard to work with preferences directly, so we want to **represent** them with **utility functions.** Given a preference relation $\succsim$, a *real-valued* utility function u represents it if the following two statements are equivalent:

$$\begin{aligned} x &\succsim y \\ &\text{and} \\ u(x) &\geq u(y) \end{aligned}$$

In other words, a utility function assigns more "points" to better alternatives, and fewer points to worse alternatives.

2. Since "bigger than or equal" $\geq$ is a complete and transitive relation on the real numbers $\mathbb{R}$ it should be clear that it can only be used to represent complete and transitive (*rational*) preferences. Thus, rationality is a **necessary** condition for **representability** (existence of utility functions representing a preference). So, whenever you can write a utility function, you have rationality (rationality is implicitly in the utility function language).

3. Technically, when we work with continuum state spaces, rationality is not **sufficient** for representability. Some discontinuous preferences (*e.g.*, lexicographic), in fact, can't be represented. For the purposes of this class, this is a non-issue, since we almost exclusively deal with continuous preferences.

4. The "points" I alluded to in the first paragraph of this section are not really points, since there is no "utility scale". This is the case, since we can't measure intensity of preference - from individual choices we can only observe if they like things more or less, not how much more, or how much less. Therefore, preference relations are **ordinal**, and only **ordinal** properties of utility functions matter. **Cardinal** properties, such as concavity/ convexity only create fake

intuition of intensity where there should be none *(there is no such thing as "twice as good", there is only "at least as good"; there is no such thing as "decreasing marginal utility" - marginal utility can only be positive, negative or zero).*

5. From the previous paragraph it follows that applying a (strictly) **monotonic transformation** to a utility function does not change the preferences. You should always apply monotonic transformations that simplify your work. But be (a bit) careful: a monotonic transformations of parts of a utility function do not have to result in a monotonic transformation of the entire utility function (the sum of the squares is not equal to the square of the sum).

6. The following ordinal properties of utility functions represent properties of preferences:

A. strictly increasing functions represent strictly monotone preferences
B. quasi-concave functions represent convex preferences
C. strictly quasi-concave functions represent strictly convex preferences