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Spring 2023

EXERCISE 1:
Public Goods

Consider an I-person economy with one private good x and one public good Q.

Assuming that each individual $i = 1, 2...I$ has and endowment of private good (only) $w^i > 0$ (you may assume $10 < w^1 < w^2 < ... < w^I$) and preferences represented by the utility function.

$$u^i(x^i, Q) = x^i + \theta_i \sqrt{Q}$$

where x_i is his/her consumption of the private good and Q is the amount of public good, $0 < \theta_1 \leq \theta_2 \leq ... \leq \theta_I \leq 2$. Let the production technology for the public good be.

$$Q = f(q) = q$$

where q is the quantity of private good used in production

a) what is the Pareto optimal level of Q (assuming both agents consume some private good)?

b) if each individual *simultaneously* contributes part of his endowment to the public good, what is the *game* that the agents play? What is the Nash equilibrium of this game (note that it is possible that one of the agents does not provide any public good in equilibrium)? Compare the equilibrium public good level Q with the level in part (a). Is the Nash equilibrium unique? (consider separately the case when $\theta_{I-1} = \theta_I$)

c) suppose that the public good provision can only be financed by a proportional tax on the agents' endowments τ ("income tax"). That is, if the tax rate is $t \in [0, 1]$ the agents will have $(1 - \tau) w^i$ units of the private good left for their consumption and the government will provide $Q = \tau \sum_{i=1}^{I} w^i$ units of the public good. What would be the ideal tax rates for each of the agents (i.e., the tax rates each one would choose if s/he were a dictator)? Graph the payoffs of a couple of agents as a function of the tax rate $\tau \in [0, 1]$.

d) assuming $\theta_i = 1$ for all i and I is an *odd* number, what would be the tax rate that cannot be defeated by a majority vote? What will be the amount of the public good Q provided? How does it compare with Nash equilibrium and Pareto efficient levels of Q ?

e) If $I = 2$ and taking $\theta_i = 1$ and $\theta_2 = 2$, $w^1 = 10, w^2 = 20$ draw in the space of utility payoffs the set of all feasible utility profiles $\left(u^1, u^2\right)$ (be careful about the case in which one of the agents has no private good)