Political Economy

Andrei Gomberg
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EXERCISE 2: Social Choice

1. Recall that May's theorem states that when the set of alternatives has only two elements, the only preference aggregation rule that on a universal domain satisfies anonymity, neutrality, monotonicity and responsiveness is the simple majority rule. Show by example that relaxing each of the last four assumptions in turn would admit other rules. Make sure your rules are defined on the universal domain.

2. Consider a two-round run-off rule. Provide an example (with three alternatives) of a monotonicity violation: i.e., a situation in which a previously winning alternative looses because some individuals switch their preferences in its favor (everything else remaining constant).

3. Recall that Arrow's theorem states that if there are at least three alternatives that every preference aggregation rule that on a universal domain always produces a complete and transitive social ranking and satisfies the weak Pareto Condition and IIA is dictatorial. Show by example that relaxing each of the assumptions in turn would admit other rules.

4. A preference aggregation rule is called oligarchic if there exists a coalition S of agents such that it is decisive (i.e., if for every $i \in S$, $x \succ^i y$ than the society ranks x strictly above y)and each individual member of the coalition has a veto (an agent is said to have a veto if as long as $x \succ^i y$ the society cannot rank y strictly above x). Such a coalition is called an olilgarchy.

a) Are dictatorial rules oligarchic?

b) Is majority rule oligarchic? What about q-supermajority rules discussed in class?

c) Is unanimity rule (i.e., x is ranked strictly above y if and only if $x \succ^i y$ for every member of the society; it is also called the Pareto rule) oligarchic?

d) Consider the voting rule used in the UN Security council (if you don't know what it is, find out). Is it oligarchic?

5. We may further relax quasi-transitivity to *acyclicity*. A preference aggregation rule is called acyclic if $x_1 P(\succeq) x_2, x_2 P(\succeq) x_3, ..., x_{n-1} P(\succeq) x_n$ implies $x_1 R(\succeq) x_n$.

a) show that quasi-transitivity implies acyclicity, but the converse is not true.

b) does the UN Security Council voting rule satisfy acyclicity?

c) until 1965 the Security council, in addition to 5 permanent members, had 6 non-permanent members and 7 votes were necessary to adopt a resolution. Was this rule acyclic? Was it quasi-transitive?