Political Economy

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EXERCISE 3: Elections

1. Consider the spatial model of political competition with sincere voters with Euclidean preferences

$$u(x,\alpha) = -|x-\alpha|$$

with ideal points α uniformly distributed on the $[0,1]$ interval and the policiy proposals $x \in [0,1]$. Parties can commit to policy by nominating candidates, so the party policy proposals are credible.

a) if political parties are Downsian (that is they only care about being elected) and the electoral system is proportional (that is parties power is roughly proportional to vote share they get), where should they locate in Nash equilibrium (if it exists) if there are, respectively, two, three, or four parties?

b) repeat the same exercise if political parties only care about forming the government, assuming the government is formed for sure by the party that gains most votes, or with equal probability by any of the parties that tie for the highest number of votes ("first past the post", FPTP).

c) repeat the same exercise if the party that forms the government is the one that gets the majority of the vote in the second-round election, in which only the parties that got the two largest vote shares in the first rounds are allowed to participate.

d) consider now the electoral system once used in Chile (well, kind of), in which each voter has a single vote and the top TWO candidates get elected together. What is the Nash equilibrium for the case of THREE parties?

e) suppose there are three parties, but one of the parties cannot (perhaps because of its reputation) choose any policy that is to the right of $\frac{1}{4}$ (that is, ithas to choose some policy $x \in \left[0, \frac{1}{4}\right]$. The other two parties are free to choose any policy. Is there a Nash equilibrium in the FPTP game? If yes, what is it, if no, why?

2. Continue working in the spatial model. Suppose now that parties care about ideology; i.e. each party, like the voters, has Euclidean preferences over the $[0,1]$ policy space However, parties can still commit to policies by nominating candidates, so that a policy proposal is taken by the voters as credible

a) Let the favorite policy of party 1 be L and the favorite party of party two be equal to R ($0 < L < \frac{1}{2} < R < 1$) and suppose the voting system is proportional and the policy implemented is the weighted average of party policy proposals, where the weights are equal to party vote shares. Suppose, further, that the voters are uniformly distributed over the $[0, 1]$ interval. What policies will the parties propose and what policy will be implemented?

b) Discuss how your answer would change instead of having a uniform distribution, you had an inverted U-shaped one (i.e., if most agents were concentrated around the median, with thin tails).

3. Consider a stochastic version of the spatial model with two parties in a FPTP environment, in which a voter $\alpha \in [0, 1]$ receives utility

$$u(x_1, \alpha) = -(x_1 - \alpha)^2$$

if she votes for party 1 proposing $x_1 \in [0, 1]$ and receives

$$u(x_2, \alpha) = -(x_2 - \alpha)^2 + \delta_\alpha + \varepsilon$$

if she votes for party 2 proposing $x_2 \in [0, 1]$. Assume that the voters are distributed over the $[0, 1]$ with some distribution function F and density f, the aggregate preference shock ε is uniformly distributed over $\left[-\frac{1}{2}, \frac{1}{2}\right]$ and the idiosyncratic preference shock faced by the voter α is uniformly distributed over $[-b_\alpha, b_\alpha]$, for some $b_\alpha > 0$. What is the equilibrium in the game played by the two parties? How does increasing b_α for a small range of alphas change it?

4. Consider a citizen-candidate model discussed in class. Assuming we are in the same spatial model as before and that the citizen ideal points are uniformly distributed in a $[0, 1]$ interval. Each citizen can decide to enter the election (become a candidate), and a candidate $\alpha \in [0, 1]$ utility function is

$$u(x, \alpha) = \begin{cases} b - c & \text{if s/he enters and wins} \\ -|x - \alpha| - c & \text{if s/he enters and looses} \\ -|x - \alpha| & \text{if s/he doesn't enter} \end{cases}$$

where x is the ideal policy of the winner (as before, ties broken randomly). Assume a large negative payoff for everyone if nobody enters.

For $b = \frac{1}{2}$ and $c = \frac{1}{4}$ describe one- and two-candidate equilibria for the FPTP and two-round run-off case.

5. Early in the 20th century there was a major debate in France about the electoral system. At the time the electoral system in use for the lower house of parliament was the two-round run-off in single-member districts (similar to the one in use today): the two front-runners in the first round competed against each other in the second round in order to determine who will represent a geographical constituency. Some people, however, argued for a shift to a proportional representation, in which seats from a multi-member district would be allocated between candidate lists based on the share of the votes obtained by each list. At the time the three major groupings in French politics were (leftist) Socialists, (centrist) Radicals and Republicans and the (rightist) Monarchists and Conservatives. Individual politicians were generally known for their adherence to these groups, but these were not pre-electoral "parties" in the modern sense: a politician would simply decide whether to register for an election without going through any partisan nomination procedure (all of this is a simplified, but, basically, a historical description). Based on the theories we discussed in class, could you guess, which of the three main groups advocated which electoral system? Explain your answer using a model or models.