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Modelación Bayesiana: Conectando R con WinBUGS

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Orden de la presentación

- Introducción al análisis Bayesiano.
- MCMC
- El proyecto BUGS
- WinBUGS
- Interface R WinBUGS
- Ejemplos
- Comentarios finales

Introducción al análisis Bayesiano

- Distribución inicial de los parámetros

$$P(\boldsymbol{\theta})$$

  Iniciales Informativas y No Informativas

- Distribución final

$$P(\boldsymbol{\theta}|\mathbf{D})$$

- Teorema de Bayes

$$P(\boldsymbol{\theta}|\mathbf{D}) \frac{L(\boldsymbol{\theta}|\mathbf{D})\, P(\boldsymbol{\theta})}{\int L(\boldsymbol{\theta}|\mathbf{D})\, P(\boldsymbol{\theta})\, d\boldsymbol{\theta}}$$

... continúa: Introducción al análisis Bayesiano

- Inferencias basadas en la distribución final
  - Media
  - Mediana
  - Moda
  - Cuantiles
  - Intervalos de probabilidad
  - Distribuciones marginales
  - Regiones de confianza
- Simulación de la distribución final
  - Métodos que simulan variables independientes
    - Inversión
    - Aceptación y rechazo
    - Muestreo por importancia y SIR

... continúa: Introducción al análisis Bayesiano

Métodos que simulan variables no independientes: MCMC

-Idea: Construir una cadena de Markov (fácil de simular) cuya distribución de equilibrio corresponda a nuestra distribución final

-Propiedades de la cadena: Homogénea, Irreducible, Aperiódica, Reversible.

- Metropolis-Hastings
- Gibbs Sampler
- Slice Sampler
- Adaptativo

No siempre es fácil implementar estos algoritmos. Aquí es donde "entra" WINBUGS

Proyecto BUGS

- **1989** inicia con una versión para UNIX llamada BUGS (**B**ayesian inference **U**sing **G**ibbs **S**ampling)
- **1998** Primera versión para Windows: Nace WinBUGS **Win**dows **B**ayesian inference **U**sing **G**ibbs **S**ampling
- Desarrollado inicialmente por la unidad de Bioestadística del **Medical Research Council (MRC)**, en Cambridge
- Actualmente, trabajando conjuntamente con el **Imperial College School of Medicine**, en St Mary's, Londres

...continúa Proyecto BUGS

- También actual: **OpenBUGS**, desarrollado en la Universidad de Helsinki, Finlandia

Los responsables

Who?

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Freely downloadable from: http://www.mrc-bsu.cam.ac.uk/bugs/winbugs/contents.shtml

WinBUGS

- Qué es WinBUGS?: Sofware para hacer análisis Bayesiano de modelos estadísticos complejos, usando Métodos Monte Carlo de Cadenas de Markov (MCMC)
- Plataforma: Windows.

  Esfuerzos para correrlo bajo Linux usando Wine y VMWare
- Métodos para muestrear en WinBUGS

  - Gibbs sampling como base para construir las cadenas
  - En cada iteración de Gibbs, se muestrea de una de las distribuciones condicionales completas
  - En general se muestrea sobre nodos univariados, pero puede hacerse de nodos multivariados

...continúa WinBUGS

- Elección del método para muestrear
  - Durante el proceso de compilación, WinBUGS elige el método que utilizará para muestrear de cada condicional completa
- Condicionales completas conjugadas $\Rightarrow$ Algoritmos estándar
- Condicionales completas no conjugadas con rango $\Re \Rightarrow$ RWMetropolis
- Condicionales completas no estándar, pero log-cóncavas $\Rightarrow$ Método de muestreo adaptativo de regiones, libre de derivación
- Condicionales completas no log-cóncavas o con rango restringido $\Rightarrow$ Slice sampling

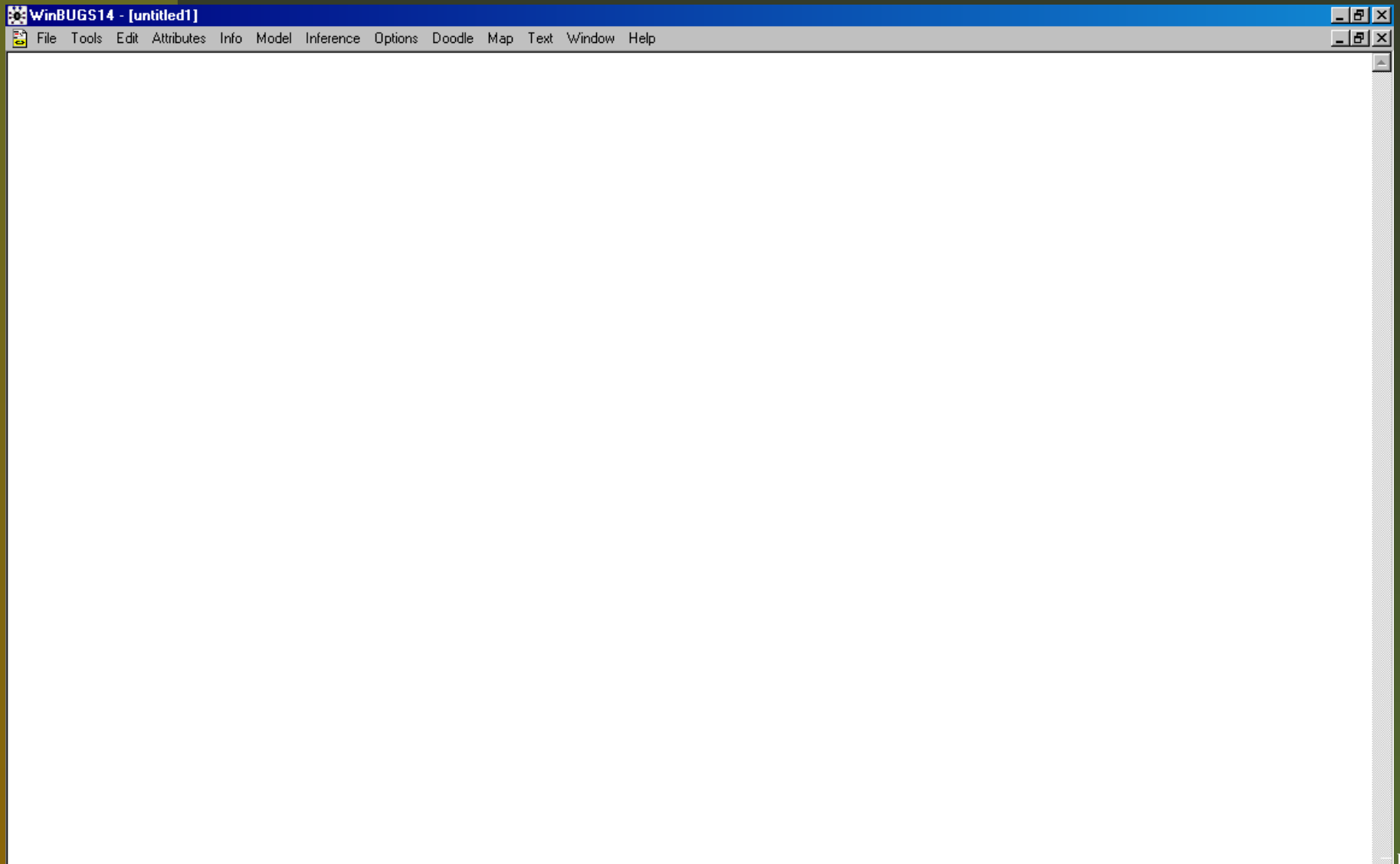
...continúa WinBUGS

Elementos para especificar un modelo en WinBUGS

- Código con el modelo a simular
- Distribuciones iniciales de los parámetros (nodos)
- Datos
- Valores iniciales de los parámetros

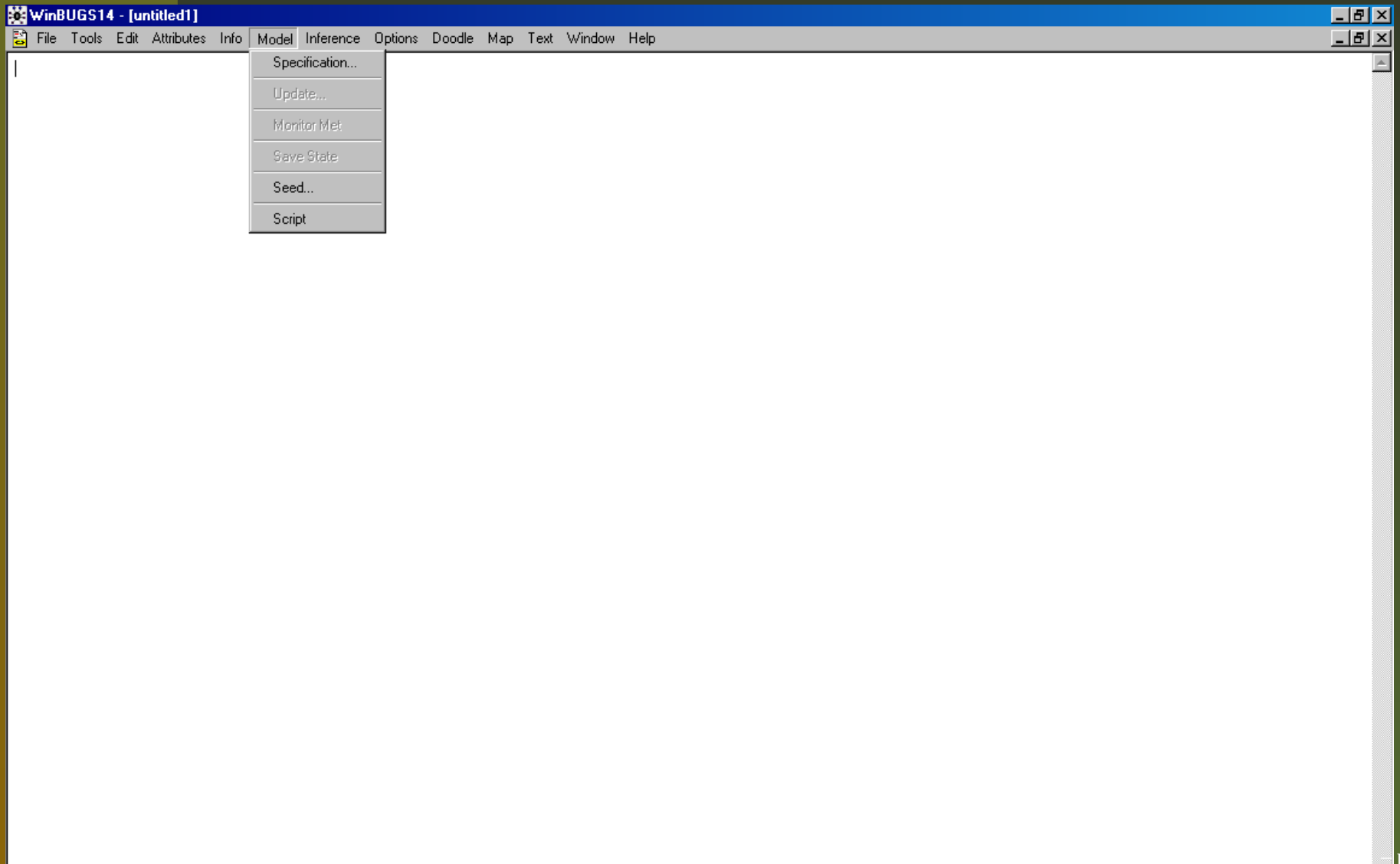
...continúa WinBUGS

Pantalla principal WinBUGS



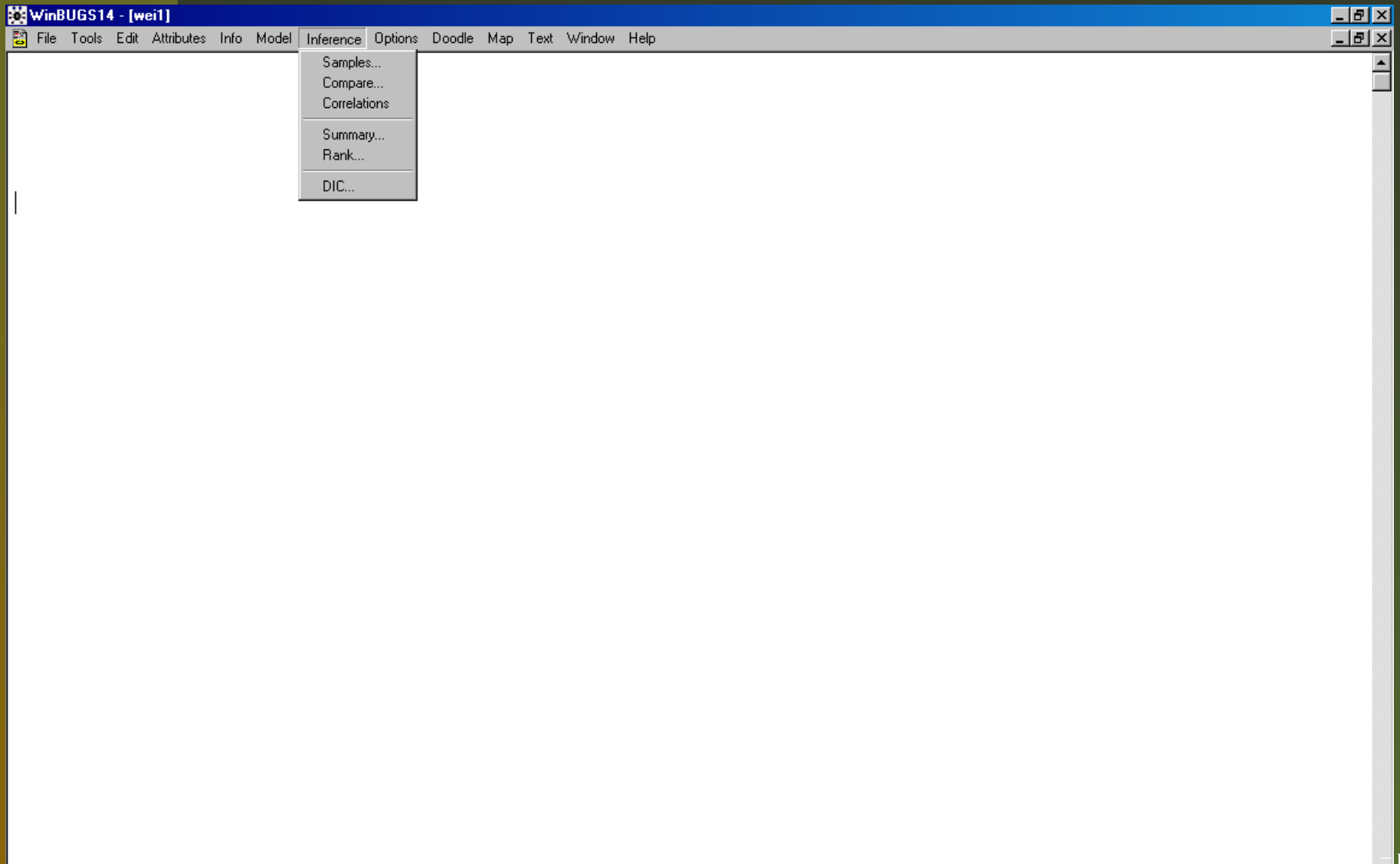
...continúa WinBUGS

Pantalla: lectura y compilación del modelo



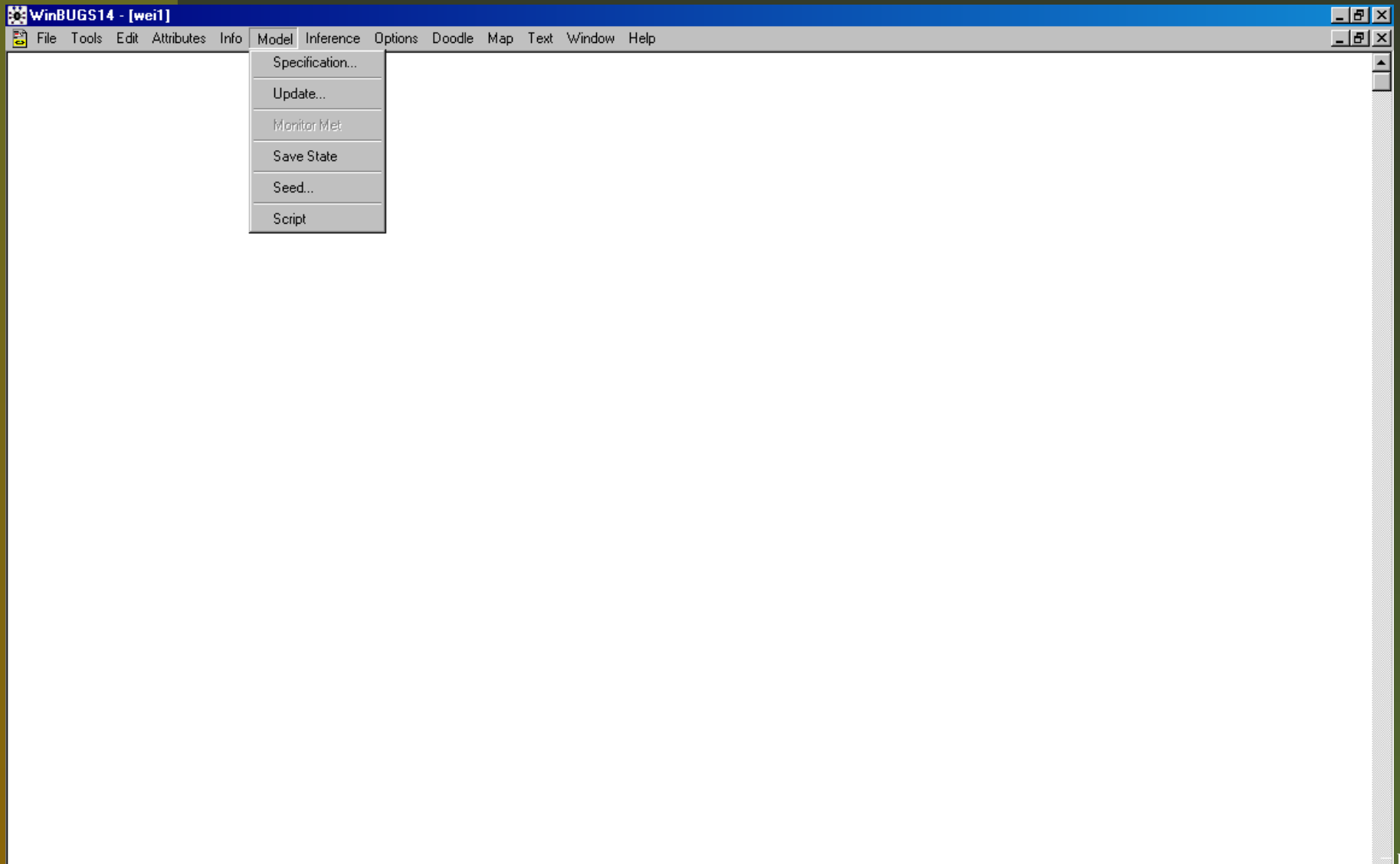
...continúa WinBUGS

Pantalla: parámetros a muestrear y especificación de la cadena



...continúa WinBUGS

Pantalla: Actualización de los parámetros



...continúa WinBUGS

Modelo de regresión lineal

Promedio de nacimientos de polluelos de cigüeñas

WinBUGS14 - [reglineal]

File Tools Edit Attributes Info Model Inference Options Doodle Map Text Window Help

```
#El modelo
model
      {
for (i in 1:n)  {
       y[i]~dnorm(mu[i],tau)
       mu[i]<-beta1+beta2*T[i]+beta3*LL[i]
 }
beta1~dnorm(0,0.001)
beta2~dnorm(0,0.001)
beta3~dnorm(0,0.001)
tau~dgamma(0.001,0.001)

sigma<-1/tau

}

#Los datos

list(n=23,
y=c(2.55,1.85,2.05,2.88,3.13,2.21,2.43,2.69,2.55,2.84,2.47,2.69
,2.52,2.31,2.07,2.35,2.98,1.98,2.53,2.21,2.62,1.78,2.30),
T=c(15.1,13.3,15.3,13.3,14.6,15.6,13.1,13.1,15.0,11.7,15.3,14.4
,14.4,12.7,11.7,11.9,15.9,13.4,14.0,13.9,12.9,15.1,13.0),
LL=c(67,52,88,61,32,36,72,43,92,32,86,28,57,55,66,26,28,96,48,90,86,78,87))

#Valores iniciales

list(beta1=0,beta2=0,beta3=0,tau=1)
```

R2WinBUGS

Paquete: R2WinBUGS

- Creadores: Sibylle Sturtz, Uwe Ligges y Andrew Gelman (2005)
- Versión 1.4 de WinBUGS en adelante
- Herramienta para llamar WinBUGS desde R
- Ventaja principal: Manipular los resultados desde R
- Conectados a Internet: install.packages(“R2WinBUGS”)
- library(“R2WinBUGS”)

...continúa R2WinBUGS

R2WinBUGS: Modelo de regresión lineal

WinBUGS14 - [RWreglin]

File Tools Edit Attributes Info Model Inference Options Doodle Map Text Window Help

```
#El modelo

model
      {
for (i in 1:n) {
       y[i]~dnorm(mu[i],tau)
       mu[i]<-beta1+beta2*T[i]+beta3*LL[i]
 }
beta1~dnorm(0,0.001)
beta2~dnorm(0,0.001)
beta3~dnorm(0,0.001)
tau~dgamma(0.001,0.001)

sigma<-1/tau

}
```

...continúa R2WinBUGS

R2WinBUGS: Modelo de regresión lineal

WinBUGS14 - [RWreglin]

File Tools Edit Attributes Info Model Inference Options Doodle Map Text Window Help

```
#El modelo

model
      {
for (i in 1:n) {
       y[i]~dnorm(mu[i],tau)
       mu[i]<-beta1+beta2*T[i]+beta3*LL[i]
 }
beta1~dnorm(0,0.001)
beta2~dnorm(0,0.001)
beta3~dnorm(0,0.001)
tau~dgamma(0.001,0.001)

sigma<-1/tau

}
```

...continúa R2WinBUGS

código para llamar desde R a WinBUGS

RGui - [C:\Documents and Settings\Administrador\Escritorio\Programas_FOROXXIII\R2reg.linear - R Editor]

File Edit Packages Windows Help

```
#Interface R WinBUGS

library(R2WinBUGS)
library(coda)
library(lattice)
y<-c(2.55,1.85,2.05,2.88,3.13,2.21,2.43,2.69,2.55,2.84,2.47,2.69,2.52,2.31,2.07,2.35,2.98,
     1.98,2.53,2.21,2.62,1.78,2.30)
T<-c(15.1,13.3,15.3,13.3,14.6,15.6,13.1,13.1,15.0,11.7,15.3,14.4,14.4,12.7,11.7,11.9,15.9,
     13.4,14.0,13.9,12.9,15.1,13.0)
LL<-c(67,52,88,61,32,36,72,43,92,32,86,28,57,55,66,26,28,96,48,90,86,78,87)
n<-23
rl.dat<-list("n","y","T","LL")
rl.ini<-function(){
         list(beta1=0,beta2=0,beta3=0,tau=1)
}
rl.par<-c("beta1","beta2","beta3","tau")

rl.s=bugs(rl.dat,rl.ini,rl.par,model.file="C:/Documents and Settings/Escritorio/RWreglin.txt",
n.chains=1,n.iter=4000,n.burnin=1000,n.thin=1,bugs.directory="C:/Archivos de programa/WinBUGS14/",
working.directory=NULL,codaPkg=FALSE)

mc.pwm<-mcmc(pwm.s$sims.array[,1,],start=1,end=1000)
IC<-HPDinterval(mc.pwm[,-14],prob=0.95)
TIC<-HPDinterval(1/mc.pwm[,1],prob=0.95)

win.graph()
par(mfrow=c(2,2))
ts.plot(beta1,type="l")
ts.plot(beta2,type="l")
ts.plot(beta3,type="l")
ts.plot(sigma,type="l")

win.graph()
par(mfrow=c(2,2))
plot(density(sb$beta[,1]))
plot(density(sb$beta[,2]))
```

...continúa R2WinBUGS

Cómo correr modelos generales

- Truco: “Ones trick” para escribir la log-verosimilitud
- Variable auxiliar de ceros
- La observación de un cero en una Poisson(λ), tiene probabilidad $\exp\{-\lambda\}$
- Si “nuestros datos observados” son un conjunto de ceros y $\lambda = -\log(L[y_i, \theta])$. Muestreando de esta Poisson, obtendremos la contribución correcta de cada observación a la verosimilitud
- También se puede hacer muestreando “unos” de una Bernoulli

...continúa R2WinBUGS

código para un modelo Weibull

WinBUGS14 - [WEIBULLMODEL]

File Tools Edit Attributes Info Model Inference Options Doodle Map Text Window Help

```
model
   {
      for(i in 1 : n) {
            t[i] ~ dweib(r,mu[i])I(cen[i],)
            mu[i] <- exp(beta0 + beta1*tra[i])
            lamda[i] <- log(mu[i])
         }
         beta0 ~ dnorm(0.0, 0.001)
         beta1 ~ dnorm(0.0, 0.001)
         r ~ dexp(0.01)
}
list(n=255,t  =
c(1.57808,1.48219,NA,2.23288,NA,3.27671,NA,1.66575,0.94247,1.6
767,2.34247,0.89863,NA,NA,0.52603,1.82192,0.93425,NA,3.35068,8.
67397,0.41096,2.78630,2.56438,NA,0.56986,NA,NA,4.38630,NA,NA,
0.86575,NA,1.15616,NA,3.13151,NA,NA,4.59452,2.88219,0.89589,1.7
6164,NA,NA,2.62192,0.16164,NA,1.52603,5.30959,0.87123,0.41644,4
.24110,0.13699,7.07671,0.13151,NA,NA,1.29863,1.29041,NA,NA,NA,2.32877,0.56438,
5.62740,1.23014,NA,5.06301,3.27671,NA,0.65753,0.84110,NA,0.1835
6,2.62466,NA,NA,0.22192,2.33973,0.52329,NA,NA,0.64110,0.38356,N
A,0.51781,NA,NA,4.42740,0.88493,2.78356,2.64658,NA,NA,0.99726,5.88493,0.41644,
3.53699,NA,NA,0.27671,0.76986,NA,NA,0.64110,1.14521,2.01644,2.8
4384,NA,1.27397,NA,2.04110,0.83562,0.92329,0.07397,NA,2.07671,N
A,NA,NA,3.30685,0.36164,1.97808,1.23836,0.10685,NA,2.06301,NA,0.50959,
0.65753,NA,NA,6.01096,0.33699,NA,0.94795,2.91781,1.59726,0.8493
2,1.38356,3.81644,NA,NA,1.00274,NA,1.18082,0.97534,2.16712,NA,1
.38356,1.71507,0.79452,NA,NA,0.42466,0.98630,NA,3.80000,NA,NA,NA,0.56164,
2.67123,1.56712,2.07397,0.33973,3.37808,3.15068,NA,3.20822,0.62
740,1.64384,1.40822,NA,1.66301,1.36986,5.46849,0.42740,1.13973,
1.73699,NA,0.85205,0.43014,1.20822,4.36164,0.52877,NA,2.89863,NA,1.21644,
NA,NA,NA,1.13699,1.69589,NA,NA,3.04932,NA,0.72603,0.73425,1.479
45,0.37808,NA,1.48219,NA,NA,1.40548,NA,NA,0.29041,NA,NA,5.16712
,NA,NA,0.53425,NA,3.59726,NA,1.78630,0.70411,NA,NA,4.32877,1.16164,NA,NA,1.41096,
NA,NA,0.98904,0.36438,NA,0.77260,4.90959,1.26849,0.58082,NA,NA,
NA,NA,1.41918,0.44110,NA,NA,NA,4.50137,3.92329,NA,0.52603,2.10685,NA,3.39178,NA,NA),
cen =
c(0.00000,0.00000,7.33425,0.00000,9.38356,0.00000,9.64384,0.00000
,0.00000,0.00000,0.00000,0.00000,9.03288,9.63014,0.00000,0.0000
0,0.00000,8.98630,0.00000,0.00000,0.00000,0.00000,0.00000,8.75342,
0.00000,8.40000,7.25205,0.00000,8.36712,8.99178,0.00000,4.76986
,0.00000,7.28767,0.00000,8.55068,8.45753,0.00000,0.00000,0.0000
0,0.00000,7.81370,8.33425,0.00000,0.00000,8.24658,0.00000,0.00000,0.00000,
0.00000,0.00000,0.00000,0.00000,0.00000,8.02740,6.16164,0.00000
,0.00000,7.99726,8.34795,7.30137,0.00000,0.00000,0.00000,0.0000
0,7.94521,0.00000,0.00000,0.60822,0.00000,0.00000,8.40000,0.00000,0.00000,
7.96438,7.77808,0.00000,0.00000,0.00000,8.04110,7.83288,0.00000
,0.00000,7.82192,0.00000,8.09863,8.16712,0.00000,0.00000,0.0000
0,0.00000,8.21370,7.41918,0.00000,0.00000,0.00000,0.00000,7.56164,7.53151,
```

Fill with a color gradient

...continúa R2WinBUGS

código modelo Weibull mediante “Ones trick”

WinBUGS14 - [R2weimodel.txt]

File Tools Edit Attributes Info Model Inference Options Doodle Map Text Window Help

```
#Modelo Weibull

model{

cte<-100000

    for(i in 1:n){

L[i]<-cen[i]*(log(r)+beta0+beta1*tra[i]+(r-1)*log(t[i]))-exp(beta0+beta1*tra[i])*pow(t[i],r)

                zeros[i]<-0
        phi[i]<- -L[i]+cte
        zeros[i]~dpois(phi[i])


}

beta0~dnorm(0,0.001)
beta1~dnorm(0,0.001)
r~dexp(0.01)

}
```

...continúa R2WinBUGS

Llamando...que es gerundio...modelo Weibull desde R

RGui - [C:\Documents and Settings\Administrador\Escritorio\Programas_FOROXXIII\R2Weimodel.r - R Editor]

File Edit Packages Windows Help

```
# Modelo Weibull
library(R2WinBUGS)
library(coda)
library(lattice)

t=c(1.57808,1.48219,7.33425,2.23288,9.38356,3.27671,9.64384,1.66575,0.94247,1.68767,2.34247,0.89863,9.03288,9.63014,0.52603,1.82192,0.93425,
8.98630,3.35068,8.67397,0.41096,2.78630,2.56438,8.75342,0.56986,8.40000,7.25205,4.38630,8.36712,8.99178,0.86575,4.76986,1.15616,7.28767,
3.13151,8.55068,8.45753,4.59452,2.88219,0.89589,1.76164,7.81370,8.33425,2.62192,0.16164,8.24658,1.52603,5.30959,0.87123,0.41644,4.24110,
0.13699,7.07671,0.13151,8.02740,6.16164,1.29863,1.29041,7.99726,8.34795,7.30137,2.32877,0.56438,5.62740,1.23014,7.94521,5.06301,3.27671,
0.60822,0.65753,0.84110,8.40000,0.18356,2.62466,7.96438,7.77808,0.22192,2.33973,0.52329,8.04110,7.83288,0.64110,0.38356,7.82192,0.51781,
8.09863,8.16712,4.42740,0.88493,2.78356,2.64658,8.21370,7.41918,0.99726,5.88493,0.41644,3.53699,7.56164,7.53151,0.27671,0.76986,7.62192,
7.79726,0.64110,1.14521,2.01644,2.84384,7.00000,1.27397,7.09589,2.04110,0.83562,0.92329,0.07397,7.30685,2.07671,7.70959,6.15890,6.89315,
3.30685,0.36164,1.97808,1.23836,0.10685,7.63836,2.06301,7.42466,0.50959,0.65753,6.93151,7.23288,6.01096,0.33699,6.47123,0.94795,2.91781,
1.59726,0.84932,1.38356,3.81644,7.06849,7.04110,1.00274,6.34795,1.18082,0.97534,2.16712,6.85479,1.38356,1.71507,0.79452,6.86301,6.50411,
0.42466,0.98630,6.13699,3.80000,6.48493,6.96438,6.78082,0.56164,2.67123,1.56712,2.07397,0.33973,3.37808,3.15068,6.81096,3.20822,0.62740,
1.64384,1.40822,6.06575,1.66301,1.36986,5.46849,0.42740,1.13973,1.73699,5.54521,0.85205,0.43014,1.20822,4.36164,0.52877,6.51507,2.89863,
6.20274,1.21644,6.00000,6.25479,6.49863,1.13699,1.69589,6.41096,6.02192,3.04932,5.62740,0.72603,0.73425,1.47945,0.37808,5.75890,1.48219,
5.88493,1.80274,1.40548,4.74795,5.24658,0.29041,5.83836,5.32055,5.16712,5.59178,5.77808,0.53425,2.22466,3.59726,5.32329,1.78630,0.70411,
4.94795,5.45479,4.32877,1.16164,5.20274,4.40822,1.41096,4.92877,5.42192,0.98904,0.36438,4.38082,0.77260,4.90959,1.26849,0.58082,4.95616,
5.12329,4.74795,4.90685,1.41918,0.44110,4.29863,4.63836,4.81370,4.50137,3.92329,4.86027,0.52603,2.10685,4.24384,3.39178,4.36164,4.81918)

cen=c(1, 1, 0, 1, 0, 1, 0, 1, 1, 1, 1, 1, 0, 0, 1, 1, 1, 0, 1, 1, 1, 1, 1, 0, 1, 0, 0, 1, 0, 0, 1, 0, 1, 0, 1, 0, 0, 1, 1, 1, 1, 0, 0, 1,
1, 0, 1, 1, 1, 1, 1, 1, 1, 1, 0, 0, 1, 1, 0, 0, 0, 1, 1, 1, 1, 0, 1, 1,0, 1, 1, 0, 1, 1, 0, 0, 1, 1, 1, 0, 0, 1, 1, 0, 1, 0, 0, 1, 1, 1, 1,
0, 0, 1, 1, 1, 1, 0, 0, 1, 1,0, 0, 1, 1, 1, 1, 0, 1, 0, 1, 1, 1, 1, 0, 1, 0, 0, 0, 1, 1, 1, 1, 1, 0, 1, 0, 1, 1, 0, 0, 1, 1, 0, 1, 1,
1, 1, 1, 1, 0, 0, 1, 0, 1, 1, 1, 0, 1, 1, 1, 0, 0, 1, 1, 0, 1, 0, 0, 0, 1, 1, 1, 1, 1, 1, 1, 0, 1, 1, 1, 1, 0, 1, 1, 1, 1, 1, 1, 0, 1, 1, 1,
1, 1, 0, 1, 0, 1, 0, 0, 0, 1, 1, 0, 0, 1, 0, 1, 1, 1, 1, 0, 1,0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 1, 0, 1, 1, 0, 0, 1, 1, 0, 0, 1, 0, 0,
1, 1, 0, 1, 1, 1, 1, 0, 0, 0, 0, 1, 1, 0, 0, 0, 1, 1, 0, 1, 1, 0, 1, 0, 0)

tra=c(1,1,0,0,1,0,0,1,0,1,1,0,1,1,0,1,0,1,0,0,1,0,0,1,1,0,0,1,1,1,1,0,1,0,0,1,1,0,1,0,1,0,1,0,1,0,0,0,1,0,0,0,1,1,0,1,1,1,0,0,1,1,0,1,0,0,1,
1,0,0,0,0,0,0,1,1,1,0,1,0,0,1,1,0,0,0,0,1,0,0,0,1,0,0,1,0,1,1,1,1,0,1,0,1,1,1,0,0,0,0,0,1,1,1,0,1,0,1,1,0,1,1,1,0,1,1,0,1,0,1,1,0,0,0,0,1,1,0,0,1,1,1,
1,1,0,1,1,0,0,1,1,0,0,0,1,1,0,1,0,0,0,1,0,0,1,1,0,1,1,1,0,1,0,0,1,1)

n<-255
w.dat<-list("n","t","cen","tra")
w.ini<-function(){list(beta0=0,beta1=0,r=1)}

w.parameters<-c("beta0","beta1","r")
w.s=bugs(w.dat,w.ini,w.parameters,model.file="C:/ERRE/R2weimodel.txt",
n.chains=1,n.iter=10000,n.burnin=5000,n.thin=1,bugs.directory="C:/Program Files/WinBUGS14/",working.directory=NULL,codaPkg=FALSE)
```

Comentarios finales

- WinBUGS ha recibido una gran cantidad de críticas porque no siempre funciona
- Mucha gente se “aterra” porque funciona como “caja negra”
- La pantalla que muestra cuando hay problemas en la simulación del modelo, ES REALMENTE INENTENDIBLE
- No obstante, hay que decir en justicia, que cuando funciona, FUNCIONA MUY BIEN!!!

“El final se acerca ya...”

Si te convenciste...

HAZ BAYESIANA