Cálculo de Probabilidades II
Respuestas Tema 1

1. Sea X = Num. max de puntos y Y = Suma de puntos

$$f_{X,Y} = \begin{cases} (1,2) & \text{c.p. } \frac{1}{36}, \\ (2,3) & \text{c.p. } \frac{2}{36}, \\ (2,4) & \text{c.p. } \frac{1}{36}, \\ (3,4) & \text{c.p. } \frac{2}{36}, \\ (3,5) & \text{c.p. } \frac{2}{36}, \\ (3,6) & \text{c.p. } \frac{1}{36}, \\ (4,5) & \text{c.p. } \frac{2}{36}, \\ (4,6) & \text{c.p. } \frac{2}{36}, \\ (4,7) & \text{c.p. } \frac{2}{36}, \\ (4,8) & \text{c.p. } \frac{1}{36}, \\ (5,6) & \text{c.p. } \frac{2}{36}, \\ (5,7) & \text{c.p. } \frac{2}{36}, \\ (5,8) & \text{c.p. } \frac{2}{36}, \\ (5,9) & \text{c.p. } \frac{2}{36}, \\ (5,10) & \text{c.p. } \frac{1}{36}, \\ (6,7) & \text{c.p. } \frac{2}{36}, \\ (6,8) & \text{c.p. } \frac{2}{36}, \\ (6,9) & \text{c.p. } \frac{2}{36}, \\ (6,10) & \text{c.p. } \frac{2}{36}, \\ (6,11) & \text{c.p. } \frac{2}{36}, \\ (6,12) & \text{c.p. } \frac{1}{36}. \end{cases}$$

2. Si

$$X|P \sim Bin(n,p) \qquad p \sim U(0,1)$$

(a)

$$\begin{aligned} f_X &= \int f(x|p)f(p)dp \\ &= \int_0^1 (\overset{n}{x})p^x(1-p)^{n-x}dp \quad I_{[0,1,2,..,n]}(x) \\ &= \binom{n}{x} I_{[0,1,2,..,n]}(x) \int_0^1 \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} p^x(1-p)^{n-x}dp \\ &= \binom{n}{x} I_{[0,1,2,..,n]}(x) \quad Beta(x+1, n-x-1) \end{aligned}$$

(b)

$$E(X) = \frac{n}{2}$$

(c)

$$V(X) = \frac{n(2n+1)}{6}$$

3. Sea

$$f_{(X,Y,Z)} = c(x+y) * z \quad I_{[,2,3,4]}(x) \quad I_{[1,2,3,4,5]}(y) \quad I_{[1,2]}(z)$$

Obtener C.

$$c\sum_x \sum_y \sum_z f_{(X,Y,Z)} = 1$$

$$\rightarrow c = \frac{1}{316}$$

4.

$$H \sim U(0,60) \quad M \sim U(0,60)$$

P(M > H + 10) = P (M - H > 10) = 1 - P(M - H ≤ 10)

$$\begin{aligned} Z = M - H \qquad & \leftrightarrow H = W - Z \\ M = M \qquad & \leftrightarrow M = W \end{aligned}$$

$$|\mathbf{J}| = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\rightarrow |\mathbf{J}| = 1$$

$$\begin{aligned} f_{(Z,W)} &= f_{(M,H)}(w, w-z) \\ &= \frac{1}{60} I_{(0,60)}(z) \quad \frac{1}{60} I_{(z,60)}(w) \end{aligned}$$

$$\begin{aligned} f_Z &= \int_z^6 0 \frac{1}{60^2} dw I_{(0,60)}(z) \\ &= \frac{1}{60}(1 - \frac{z}{60}) \end{aligned}$$

$$1 - P(M - H \leq 10) = 1 - \frac{1}{60}(1 - \frac{z}{60})$$

5. Si $f_{(X,Y)} = 24$ xy $I_{(0,1)}(x)\ I_{(0,1)}(y)\ I_{(0,1)}(x+y)$, entonces, X y Y no son variables aleatorias. Simplemente porque la indicadora depende de ambras variables.

6. Tabla

$f_{(Y,X)}$	0	1	2	3	4	f_x
0	$\frac{187}{3200}$	$\frac{300}{3200}$	$\frac{150}{3200}$	$\frac{225}{3200}$	$\frac{338}{3200}$	$\frac{1200}{3200}$
1	$\frac{156}{3200}$	$\frac{250}{3200}$	$\frac{125}{3200}$	$\frac{189}{3200}$	$\frac{280}{3200}$	$\frac{1000}{3200}$
2	$\frac{125}{3200}$	$\frac{200}{3200}$	$\frac{100}{3200}$	$\frac{150}{3200}$	$\frac{225}{3200}$	$\frac{800}{3200}$
3	$\frac{32}{3200}$	$\frac{50}{3200}$	$\frac{25}{3200}$	$\frac{36}{3200}$	$\frac{57}{3200}$	$\frac{200}{3200}$
f_Y	$\frac{500}{3200}$	$\frac{800}{3200}$	$\frac{400}{3200}$	$\frac{600}{3200}$	$\frac{900}{3200}$	1

P(Y < 2) = 0.6875
P(X < 2) = 0.40625
P(Y = 0 | X = 3) = 0.375
P(Y < 3 | X = 3) = 0.94
P(Y ≥ 3 | X = 3) = 0.06
X y Y no son independientes, $f_{(Y,X)} \neq f_X f_Y$

7. Sea

$$f_{X,Y,Z} = \frac{n!}{x!y!z!} p_x^x p_y^y (1 - p_x - p_y)^z$$

$$P(X = 2|Y = 0) = \frac{(\frac{3}{9})^2}{(\frac{3}{9})^2 + 2\frac{3}{9}\frac{2}{9}\frac{4}{9} + (\frac{2}{9})^2}$$

$$P(X + Y \leq 1) = 2 * \frac{2}{9}\frac{4}{9} + 2 * \frac{3}{9}\frac{2}{9}$$

$$P(Y \leq 1|X = 1) = \frac{2 * \frac{3}{9}\frac{2}{9} + 2 * \frac{3}{9}\frac{4}{9}}{2 * \frac{3}{9}\frac{2}{9} + 2 * \frac{3}{9}\frac{4}{9}}$$

$$F_{X,Y} = \sum_x \sum_y \sum_z^2 \frac{2!}{x!y!z!} p_x^x p_y^y (1 - p_x - p_y)^z, x + y + z = 2.$$

$$F_X = \sum_x \sum_y^2 \sum_z^2 \frac{2!}{x!y!z!} p_x^x p_y^y (1 - p_x - p_y)^z, x + y + z = 2.$$

$$F_Y = \sum_y \sum_x^2 \sum_z^2 \frac{2!}{x!y!z!} p_x^x p_y^y (1 - p_x - p_y)^z, x + y + z = 2.$$

$$\begin{aligned} F_{X|Y} &= \frac{F_{X,Y}}{F_Y} \\ &= \frac{\sum_x \sum_y \sum_z^2 \frac{2!}{x!y!z!} p_x^x p_y^y (1 - p_x - p_y)^z, x + y + z = 2}{\sum_y \sum_x^2 \sum_z^2 \frac{2!}{x!y!z!} p_x^x p_y^y (1 - p_x - p_y)^z, x + y + z = 2} \end{aligned}$$

$$f_{X|Y} = \frac{P(X = x, Y = y)}{P(Y = y)}$$

$$f_{Y|X} = \frac{P(X = x, Y = y)}{P(X = x)}$$

8. Sea

$$f_{X,Y} = cyI_{(0,2)}(x) \quad I_{(x,2)}(y) \tag{1}$$

$$\begin{aligned} 1 &= \int_0^2 \int_x^2 cy dy dx \\ \rightarrow c &= \frac{3}{8} \end{aligned}$$

$$\begin{aligned} f_X &= \int_x^2 \frac{3}{8} y \mathtt{d} y \\ &= \frac{3}{8}(2 - \frac{x^2}{2}) I_{(0,2)}(x) \end{aligned}$$

$$\begin{aligned} f_Y &= \int_0^y \frac{3}{8} y \mathtt{d} x \\ &= \frac{3}{8} y^2 I_{(0,2)}(y) \end{aligned}$$

$$\begin{aligned} F_{X,Y} &= \int_0^x \int_x^y \frac{3}{8} z \mathtt{d} z \mathtt{d} w \\ &= \frac{3}{8}(\frac{xy^2}{2} - \frac{x^2}{6}) \end{aligned}$$

$$F_{X,Y} = \begin{cases} \frac{3}{8}(\frac{xy^2}{2} - \frac{x^2}{6}) & 0 < x < y < 2, \\ \infty & 0 < y < 2, x \geq y, \\ \infty & 0 < x < 2, y \geq 2, \\ \infty & x \geq 2, y \geq 2, \\ -\infty & x \leq 0, y \leq 0. \end{cases}$$

Y como,

$$\frac{3}{8} y = f_{(X,Y)} \neq \frac{3}{8}(2 - \frac{x^2}{2}) \frac{3}{8} y^2 = f_X f_Y$$

X y Y no son independientes.

$$F_X = F_{X,Y}(x, \infty)$$
$$= \frac{3y^2}{8} - \frac{1}{2}$$

Sea

$$f_{X,Y} = ce^{-x}e^{-2y}I_{(0,\infty)}(x)I_{(0,x)}(y) \tag{2}$$

$$1 = \int_0^\infty \int_0^x ce^{-x}e^{-2y}dydx$$
$$\to c = 3$$

$$f_X = 3e^{-x}\int_0^x e^{-2y}\mathtt{d}y$$
$$= \frac{3}{2}e^{-x} - e^{-3x}I_{(0,\infty)}(x)$$

$$f_Y = 3e^{-2y}\int_y^\infty e^{-x}\mathtt{d}y$$
$$= 3e^{-3y}I_{(0,\infty)}(y)$$

$$F_{X,Y} = 3\int_0^x \int_0^y e^{-w}e^{-2z}\mathtt{d}z\mathtt{d}w$$
$$= (\frac{3}{2} - \frac{3}{2}e^{-2y})(1 - e^{-x})I^X_{(0,\infty)}I_{(0,x)}(y)$$

$$F_X = F_{X,Y}(x, \infty) = \frac{3}{2}(1 - e^{-x})I_{(0,\infty)}(x)$$

$$F_Y = F_{X,Y}(y, \infty) = \frac{3}{2}(1 - e^{-2y})I_{(0,\infty)}(y)$$

Y como

$$f_{X,Y} \neq f_X f_Y$$

X y Y no son independientes.

Sea

$$f_{X,Y} = c(x + y)I_{(0,2)}(x)I_{(0,1)}(y) \tag{3}$$

$$1 = c\int_0^2\int_0^1 c(x+y)\mathtt{d}y\mathtt{d}x$$
$$\rightarrow c = \frac{1}{3}$$

$$f_X = \frac{1}{3}\int_0^1 (x+y)\mathtt{d}y$$
$$= \frac{1}{3}(x+\frac{1}{2})I_{(0,2)}(x)$$

$$f_Y = \frac{1}{3}\int_0^2 (x+y)\mathtt{d}x$$
$$= \frac{1}{3}(2+2y)I_{(0,1)}(y)$$

$$F_{X,Y} = \frac{1}{3}\int_0^x\int_0^y (w+z)\mathtt{d}z\mathtt{d}w$$
$$= \frac{1}{3}(\frac{yx^2}{2}+\frac{xy^2}{2})I_{(0,2)}(x)I_{(0,1)}(y)$$

$$F_X = F_{X,Y}(X,1) = (\frac{x^2}{6}+\frac{x}{6})I_{(0,2)}(x)$$

$$F_Y = F_{X,Y}(2,Y) = (\frac{2y}{3}+\frac{y^2}{3})I_{(0,1)}(y)$$

Y como

$$f_{X,Y} \neq f_X f_Y$$

no son independientes.

9. Sea

$$f_{X,Y} = \frac{e^{-x/y}e^{-y}}{y}I_{(0,\infty)}(x)I_{(0,\infty)}(y)$$

$$P(X>1|Y=y) = \frac{P(X>1, Y=y)}{P(Y=y)}$$

$$
\begin{aligned}
f_Y &= P(Y=y) \\
&= \frac{e^{-y}}{y}\int_0^{\infty} e^{-x/y}\mathtt{d}x \\
&= e^{-y} I_{(0,\infty)}(y)
\end{aligned}
$$

$$
\begin{aligned}
P(X>1, Y=y) &= \frac{e^{-y}}{y}\int_1^{\infty} e^{-x/y}\mathtt{d}x \\
&= e^{-y}e^{-1/y}
\end{aligned}
$$

$$
P(X>1|Y=y) = e^{-1/y}
$$

10. Sea

$$
f_{X,Y} = \frac{15}{2}x2 - x - yI_{(0,1)}(x)I_{(0,1)}(y)
$$

$$
f_{X|Y} = \frac{f_{X,Y}}{f_Y}
$$

$$
\begin{aligned}
f_Y &= \int_0^1 \frac{15}{2}x(2-x-y)\mathtt{d}x \\
&= \frac{15}{2}(\frac{2}{3} - \frac{y}{2})I_{(0,1)}(y)
\end{aligned}
$$

$$
\rightarrow f_{X|Y} = \frac{x(2-x-y)}{\frac{2}{3}-\frac{y}{2}} I_{(0,1)}(x)
$$

11. Sea

$$
f_{X,Y} = \frac{21}{4}x^2y \quad I_{(-1,1)}(x)I_{(x^2,1)}(y)
$$

$$
P(X \leq y | X \geq 2y) = \frac{P(X \leq y, X \geq 2y)}{P(X \geq 2y)}
$$

$$
\begin{aligned}
f_X &= \frac{21}{4}\int_{x^2}^1 x^2y\mathtt{d}y \\
&= \frac{21}{8}(x^2 - x^6)I_{(-1,1)}(x)
\end{aligned}
$$

$$P(X \geq 2y) = \frac{21}{8}\int_{2y}^{1}(x^2 - X^6)\mathtt{d}x$$
$$= \frac{381}{3}y^7 - \frac{17}{8}y^3$$

$$P(X \leq y | X \geq 2y) = \frac{\frac{381}{3}y^7 - \frac{17}{8}y^3}{\frac{1}{2} - 7y^3 + 48y^7}$$

$$P(x^2 + y^2 \leq 1)$$

Sea

$$z = x^2 + y^2 \qquad \leftrightarrow y = \sqrt{w}$$
$$w = y^2 \qquad \leftrightarrow x = \sqrt{z - w}$$

$$\rightarrow |J| = \frac{1}{4\sqrt{x}\sqrt{z-w}}$$

$$f_{Z,W} = \frac{21}{16}\sqrt{z-w}I_{(0,1)}(z)I_{(0,z)}(w) + I_{(1,2)}(z)I_{(z-1,1)}(w)$$

$$\rightarrow f_Z = \int_0^z \frac{21}{16}\sqrt{z-w}\mathtt{d}wI_{(0,1)}(z) + \int_z -1^1\frac{21}{16}\sqrt{z-w}\mathtt{d}wI_{(1,2)}(z)$$
$$= \frac{7}{8}z^{3/2}I_{(0,1)}(z) + \frac{7}{8}(1 - (z-1)^{3/2})I_{(1,2)}(z)$$

$$P(x^2 + y^2 \leq 1) = \int_0^1 \frac{7}{8}z^{3/2}\mathtt{d}z = \frac{14}{40}$$

$$P(x^2 + y^2 \leq 1 | Y = \frac{1}{2}) = \frac{P(x^2 + y^2 \leq 1, Y = \frac{1}{2})}{P(Y = \frac{1}{2})}$$

$$f_Y = y\int_{-\sqrt{y}}^{\sqrt{y}} \frac{21}{4}x^2\mathtt{d}x \quad I_{(0,1)}(y)$$
$$= \frac{21}{6}y^{5/2}I_{(0,1)}(y)$$
$$\rightarrow P(Y = \frac{1}{2}) = 0.61781$$

$$P(x^2+y^2\leq 1, Y=\frac{1}{2}) = P(|X|\leq \sqrt{3/4})$$

$$\begin{aligned} f_X &= \int_{x^2}^{1} \frac{21}{4}x^2 y \mathtt{d}y \\ &= \frac{21}{8}(x^2-x^6) \quad I_{(-1,1)}(x) \\ \rightarrow P(|X|\leq \sqrt{3/4}) &= \int_{-\sqrt{3/4}}^{\sqrt{3/4}} \frac{21}{8}(x^2-x^6)\mathtt{d}x \\ &= 0.32862 \end{aligned}$$

$$P(x^2+y^2\leq 1|Y=\frac{1}{2}) = \frac{0.3286}{0.6187}$$

12. Sea

$$\begin{aligned} f(x_i|\mu,\sigma^2) &\sim N(x|\mu,\sigma^2) \\ f(\mu|\theta,\delta^2) &\sim N(\theta,\delta^2) \end{aligned}$$

$$\begin{aligned} f(\mu|x_i) &= \frac{f_(\mu,x_i)}{f_{x_i}} \\ f_{\mu,x_i} &= \frac{1}{\sqrt{2\pi\sigma^2}}\frac{1}{\sqrt{2\pi\delta^2}}exp-\frac{(x-\mu)^2}{2\sigma^2}exp-\frac{(x-\theta)62}{2\delta^2}I_{\Re}(\mu)I_{\Re}(x) \\ \rightarrow f_{x_i} &= \int_- \infty^\infty \frac{1}{\sqrt{2\pi\sigma^2}}\frac{1}{\sqrt{2\pi\delta^2}}exp-\frac{(x-\mu)62}{2\sigma^2}exp-\frac{(x-\theta)62}{2\delta^2}\mathtt{d}\mu I_{\Re}(x) \end{aligned}$$

$$\begin{aligned} f_{\mu|x_i} &= \frac{\frac{1}{\sqrt{2\pi\sigma^2}}\frac{1}{\sqrt{2\pi\delta^2}}exp-\frac{(x-\mu)^2}{2\sigma^2}exp-\frac{(x-\theta)62}{2\delta^2}I_{\Re}(\mu)I_{\Re}(x)}{\int_- \infty^\infty \frac{1}{\sqrt{2\pi\sigma^2}}\frac{1}{\sqrt{2\pi\delta^2}}exp-\frac{(x-\mu)^2}{2\sigma^2}exp-\frac{(x-\theta)62}{2\delta^2}\mathtt{d}\mu I_{\Re}(x)} \\ &= \frac{exp-\frac{(x-\mu)^2}{2\sigma^2}}{\int_- \infty^\infty exp-\frac{(x-\mu)^2}{2\sigma^2}\mathtt{d}\mu}I_{\Re}(\mu) \end{aligned}$$

13. Sea

$$f_{X,Y,Z} = 8xyz \quad I_{(0,1)}(x)I_{(0,1)}(y)I_{(0,1)}^Z(z)$$

(a)

$$P(X<Y<Z) = \int_0^1\int_0^z\int_0^y 8xyz = \frac{1}{3}$$

(b)

$$f_X = \int_0^1 \int_0^1 8xyz\mathtt{d}y\mathtt{d}z = 2x \quad I_{(0,1)}(x)$$
$$f_Y = \int_0^1 \int_0^1 8xyz\mathtt{d}x\mathtt{d}z = 2y \quad I_{(0,1)}(y)$$
$$f_Z = \int_0^1 \int_0^1 8xyz\mathtt{d}y\mathtt{d}x = 2z \quad I_{(0,1)}(z)$$

(c) $\rho_{YZ} = 0$, porque Y y Z son variables aleatorias independientes

14. Sea X = Num de la bola   Y = Mayor de los dos numeros seleccionados.

(a)

X/Y	1	2	3	f_x
1	0	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{6}$
2	0	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{6}$
3	0	0	$\frac{2}{6}$	$\frac{2}{6}$
f_Y	0	$\frac{2}{6}$	$\frac{4}{6}$	1

(b)

$$P(X = 1|Y = 3) = \frac{1}{4}$$

$$\rho_{XY} = \frac{Cov(X,Y)}{\sigma_X \sigma_Y}$$
$$Cov(X,Y) = E(XY) - E(X)E(Y)$$
$$E(XY) = \sum_2^3 \sum_2^3 xy = \frac{11}{2}$$
$$E(X) = 2$$
$$E(Y) = \frac{8}{3}$$
$$\sigma_X = \sqrt{2/3}$$
$$\sigma_Y = \frac{\sqrt{2}}{3}$$
$$\rightarrow \rho_{XY} = 0.43301$$

15. Sea

$$f_{XY} = \frac{1}{8}e^{-x}(x^2 - y^2) \quad I_{\Re}(x) I_{(-\infty,\infty)}(y)$$

(a)

$$P(X < 1, Y < 0) = \int_0^1 \int_{-x}^0 \frac{1}{8}e^{-x}(x^2 - y^2)\mathtt{d}y\mathtt{d}x$$
$$= (1 - \frac{8}{3}e^{-1})$$

(b)

$$f_{Y|X} = \frac{f_(X,Y)}{f_{(X)}}$$
$$= \frac{3}{4}\frac{(x^2-y^2)}{x^3} I(-\infty,\infty)(y)$$

(c)

$$\rho_{YX} = \frac{Cov(XY)}{\sigma_X \sigma_Y}$$

(d) Sea W = 2X + Y

$$E(W) = \int_0^\infty \int_{-x}^{x} (2x+y)\frac{1}{8}e^{-x}(x^2-y^2)\mathtt{d}y\mathtt{d}x$$
$$= \frac{1}{8}(24 - 65e^{-1})$$

16. Sea

$$Probabilidadtotalnmerodehijos = \begin{cases} \text{no hijos} & \text{c.p. } 0.15, \\ \text{un hijo} & \text{c.p. } 0.2, \\ \text{dos hijo} & \text{c.p. } 0.35, \\ \text{tres hijos} & \text{c.p. } 0.3. \end{cases}$$

G/B	0	1	2	3
0	0.15	0.1	0.0875	0.0375
1	0.1	0.175	0.1125	0
2	0.0875	0.1125	0	0
3	0.0375	0	0	0

17. Sea

$$f_{P|N} = \frac{f_P f_{N|P}}{f_N}$$

$$f_N = \int_0^1 \binom{n+m}{n} p^n (1-p)^m \mathtt{d}p$$
$$= \binom{n+m}{n} \frac{\Gamma(n+1)\Gamma(m+1)}{\Gamma(n+m+2)}$$

$$f_{P|N} = \frac{p^n(1-p)^m}{\frac{\Gamma(n+1)\Gamma(m+1)}{\Gamma(n+m+2)}}$$

18. Sea

$$f_{X,Y} = cxy \quad I_{(1,2,3)}(x) I_{(2,3,4)}(y)$$

(a)

$$C = \frac{1}{54}$$

(b)

$$P(X \geq 2, Y \leq 3) = \frac{1}{54} \sum_{x=2}^{3} \sum_{y=2}^{3} xy = 0.46296$$

(c)

$$f_X = \frac{x}{6} \quad I_{(1,2,3)}(x)$$
$$E(X) = \frac{7}{3}$$
$$E(X^2) = 6$$
$$Var(X) = \frac{5}{9}$$

(d)

$$Cov(XY) = E(XY) - E(X)E(Y)$$
$$E(XY) = \frac{1}{54} \sum_{x=1}^{3} \sum_{y=2}^{4} x^2 y^2 = \frac{203}{27}$$
$$f_Y = \frac{y}{9} \quad I_{(2,3,4)}^{Y}$$
$$E(Y) = \frac{29}{9}$$
$$E(Y^2) = 11$$
$$V(Y) = \frac{5}{9}$$
$$Cov(X,Y) = 0 \rightarrow \rho_{XY} = 0$$

(e) Son independientes

(f)

$$V(5X - 3Y) = E((5x - 3y)^2) - E^2(5x - 3y)$$
$$= \frac{175}{9}$$

19. Sea X = Num de puntos observados y Y = Numero de soles obtenidos

$$E(Y|X = 1) = \frac{1}{2}$$
$$E(Y|X = 2) = \frac{2}{2}$$
$$E(Y|X = 3) = \frac{3}{2}$$
$$E(Y|X) = \sum_{x=1}^{6} \frac{x}{2}$$

20. Si X$\sim$ Po(θ)

(a)

$$E(X|X \geq 2) = \frac{1}{1-F_X}\sum_{x=2}^{\infty} xe^{-\theta}\frac{\theta^x}{x!}$$
$$= \frac{1}{1-\sum_{x=0}^{2} xe^{-\theta}\frac{\theta^x}{x!}}(\theta - \theta e^{-\theta})$$

(b)

$$E(X|X \leq 4) = \frac{1}{f(X \leq 4)}\sum_{x=0}^{4} xe^{-\theta}\frac{\theta^x}{x!}$$

21. Sea Y = Num de bolas negras X = Num de bolas rojas en la muestra

(a)

$$E(X|Y) = \sum_{x=0}^{2}\frac{xf(x,y)}{f(y)} = 2$$

(b)

$$E(X) = \sum_{x=0}^{2}\sum_{y=1}^{2} xf_{XY} = \frac{8}{9}$$

(c)

$$E(X^2|Y) = \frac{8}{3}$$

(d)

$$V(X|Y) = \frac{2}{3}$$

(e)

$$E(X^2) = \frac{10}{9}$$

(f)

$$V(X) = \frac{26}{81}$$

22. Sea

$$f_X = \int_0^{\infty}\frac{\theta^{x+1}e^{-\theta(x+1)}}{(x)!}\mathtt{d}\theta \quad I_{[0,1,2,\ldots]}(x) \quad Kernel \quad de \quad una \quad Gamma(\alpha = (x{+}2), \beta = (x{+}1))$$

(a)

$$E(X) = (x+2)(x+1)$$

(b)

$$V(X) = (x+1)^2(x+3)(x+2) - ((x+2)(x+1))^2$$

23. Sea X $\sim$ Exp(θ)

(a)

$$\begin{aligned} E(X|X>a) &= \frac{1}{1-F(a)}\int_a^\infty \theta x e^{-\theta x}\mathtt{d}x \\ &= \frac{1}{1-F(a)}(ae^{-\theta a}-\frac{e^{-\theta a}}{\theta}) \end{aligned}$$

24.

$$|\rho_{XY}| = \frac{Cov(XY)}{\sigma_X\sigma_Y} = \frac{bV(X)}{bV(X)} = 1$$

25.

$$\begin{aligned} E(X) &= E_Y E(X|Y) \\ &= E_Y \int_- \infty^\infty x f(x|y)\mathtt{d}x \\ &= \int_- \infty^\infty \int_- \infty^\infty x f(x|y)\mathtt{d}x\mathtt{d}y \end{aligned}$$

$$\begin{aligned} E_Y Var(X|Y) &= E_Y E(X^2|Y) - E^2(X|Y) \\ &= E_Y(X^2|Y) - E_Y E^2(X|Y) \\ &= V(X) - E^2(X) - E_Y E^2(X|Y) \end{aligned}$$

$$\rightarrow E_Y Var(X|Y) = V(X) - V_Y(X|Y)$$

26. P.D. que

$$F_{X,Y} = 1, \quad x+y \geq 0, \quad 0 e.o.c.$$

Es una funcion de distribucion.

(a)

$$\begin{aligned} F_{X,Y}(-\infty,y) &= lim_{x\rightarrow-\infty}F_{X,Y} = 0 \\ F_{X,Y}(x,-\infty) &= lim_{y\rightarrow\infty}F_{X,Y} = 0 \\ F_{X,Y}(-\infty,\infty) &= lim_{(x,y)\rightarrow(-\infty,\infty)}F_{X,Y} = 1 \end{aligned}$$

(b) Si

$$a_1 < b_1 \quad y \quad a_2 < b_2,$$

entonces

$$P(a_1 \leq X < b_1, a_2 \leq Y < b_2) = F_{X,Y}(b_1,b_2) - F_{X,Y}(a_1,b_2) - F_{X,Y}(b_1,a_2) + F_{X,Y}(a_1,a_2) \geq 0.$$

(c) F$_{X,Y}$ (x,y) es continua por la derecha en cada argumento, ie,

$$\begin{aligned} lim_{h\to 0}F(x+h,y) &= lim_{h\to 0}F(x,y+h) \\ &= F_{X,Y}(x,y) \end{aligned}$$

27. Sea

$$f_X = \begin{cases} 1 & \text{c.p. } \frac{1}{2}, \\ 2 & \text{c.p. } \frac{1}{4}, \\ 3 & \text{c.p. } \frac{1}{4}. \end{cases}$$

(a)

$$\begin{aligned} M_X(t) &= E(e^{xt}) \\ &= \sum_{x=1}^{3} e^{xt} f(x) \\ &= \frac{1}{2}e^{t} + \frac{1}{4}e^{2t} + \frac{1}{4}e^{3t} \end{aligned}$$

(b)

$$\begin{aligned} E(x) &= M'_X(t)|_{t=0} \\ &= \frac{7}{4} \end{aligned}$$

(c)

$$E(x^2) = \frac{15}{4}$$

(d)

$$V(X) = \frac{11}{16}$$

28. Sean X y Y v.a. continuas con funcion de densidad dada por

$$f_{X,Y} = \begin{cases} \lambda^2 e^{-\lambda y} & 0 \leq x \leq y \\ 0 & \text{e.o.c.} \end{cases}$$

(a)

$$\begin{aligned} f_X &= \int_x^\infty \lambda^2 e^{-\lambda y} \mathtt{d}y \\ &= \lambda e^{-x\lambda} I_{(0,\infty)}(x) \end{aligned}$$

(b)

$$\begin{aligned} f_Y &= \int_0^y \lambda^2 e^{-\lambda y} \mathtt{d}x \\ &= \lambda^2 e^{-y\lambda} I_{(0,\infty)}(y) \end{aligned}$$

(c)

$$F_{X,Y} = \int_0^x \int_0^y \lambda^2 e^{-\lambda u} \mathtt{d}u\mathtt{v}$$
$$= \lambda x(1 - e^{-\lambda y})$$

(d)

$$f_{Y|X} = \frac{f_{X,Y}}{f_X}$$
$$= \frac{\lambda^2 e^{-\lambda y} I_{(0,\infty)}(y)}{\lambda e^{-x\lambda}}$$

29. P.D.

$$|\rho| = 1 \leftrightarrow \exists a, b \in R, b \neq 0$$

Y

$$P(Y = a + bx) = 1$$

Demostracion con la desigualdad de Cauchy Schwarz Seam x_1 y X_2 v.a.'s con segundos momentos finitos.

$$|E(X_1X_2)| \leq \sqrt{E(X_1^2 E(X_2^2)}$$

Sea

$$Y_1 = X_1 - \mu_1$$
$$Y_2 = X_2 - \mu_2$$

$$\rightarrow E(Y_1Y_2) = E((X_1 - \mu_1)(X_2 - \mu_2))$$
$$= Cov(X_1, X_2)$$

$$E(Y_1^2) = Var(X_1)$$
$$E(Y_2^2) = Var(X_2)$$

$$\rightarrow |E(Y_1Y_2)| \leq \sqrt{E(Y_1^2)E(Y_2^2)}$$
$$\leftrightarrow |Cov(X_1, X_2)| \leq \sqrt{Var(X_1)Var(X_2)}$$
$$\leftrightarrow \frac{|Cov(X_1, X_2)|}{\sqrt{Var(X_1)Var(X_2)}} \leq 1$$
$$\leftrightarrow |\rho_{X_1,X_2}| \leq 1$$

30. Sea

$$X \sim \chi^2_{(2)}$$

Y

$$\begin{aligned} W &= \frac{X_1 - X_2}{2} \\ Y &= X_1 \\ &\rightarrow |\mathbf{J}| = 2 \end{aligned}$$

$$\begin{aligned} \rightarrow f_W &= \int 2\frac{1}{2}e^{-\frac{y}{2}}\frac{1}{2}e^{-y+2w}\mathtt{d}y \\ &= \frac{1}{3}e^{2w}I_{(0,\infty)}(W) \end{aligned}$$

31. Sea

$$H = \frac{2X}{W}$$

$$\begin{aligned} X &\sim U(3,5) \\ W &\sim U(10,20) \end{aligned}$$

$$\begin{aligned} E(H) &= \int_3^5 \int_{10}^{20} \frac{2x}{w}\frac{1}{2}\frac{1}{10}\mathtt{d}w\mathtt{d}x \\ &= \frac{4ln2}{5} \end{aligned}$$

32. Si X|P $\sim$ exp(λ) $\quad\lambda \sim \exp(\theta)$

$$\begin{aligned} f_X &= \int f_{X|\lambda}f_\lambda \mathtt{d}\lambda \\ &= \int_0^\infty \theta\lambda e^{-\lambda(x+\theta)}\mathtt{d}\lambda \\ &= \frac{\theta}{(\theta+x)^2} \quad I_{(0,\infty)}(x) \end{aligned}$$

33. P.D. f$_{X,Y}$ = h(x) g(y)
Dem.

$$f_{X,Y} = f_{X|Y}f_X \quad \sim 1$$

$$\begin{aligned} f_Y &= \int f_{X,Y}\mathtt{d}x \\ &= \int f_{X|Y}f_X\mathtt{d}x \\ &porindependencia, \\ &f_{X|Y}\int f_X\mathtt{d}x \\ &= f_{X|Y} \quad \sim 2 \end{aligned}$$

Sustituyendo 1 en 2

$$f_{X|Y} = f_X f_Y$$

34. Sea

$$f_{X,Y} = 4e^{-2x}e^{-2y} \quad I_{(0,\infty)}(x)I_{(0,\infty)}(y)$$

(a)

$$P(X > 1, Y < 1) = e^{-2}(1 - e^{-2})$$

(b)

$$P(X < Y) = P(X - Y < 0)$$

Sea

$$X - Y = Z \qquad \to W = YY = W \qquad\qquad \to X = Z - W$$

$$\leftrightarrow |\mathbf{J}| = 1$$

$$\begin{aligned} f_{Z,W} &= f_{X,Y}(z - w, w) \\ &= 4e^{-2z} \quad I_{(0,\infty)}(z)I_{(0,z)}(w) \\ \to f_Z &= 2(1 - e^{-2z}) \quad I_{(0,\infty)}(z) \end{aligned}$$

$$P(X < Y) = P(Z < 0) = 0$$

(c)

$$P(X < a) = (1 - e^{-2a})$$

35. Sea

$$f_{X,Y} = 3x \quad I_{(y,1)}(x)I_{(0,x)}(y)$$

(a)

$$P(X \geq \frac{1}{2}|Y \leq 2X) = \frac{P(X \geq \frac{1}{2}, Y \leq 2X)}{P(Y \leq 2X)}$$

$$P(Y \leq 2X) = \int_0^{2x} \frac{3}{2}(1 - y^2)\mathtt{d}y$$

$$P(X \geq \frac{1}{2}, Y \leq 2X) = \int_{1/2}^{1}\int_0^{2x} 3x\mathtt{d}y\mathtt{d}x$$

(b)

$$P(X^2 + Y^2 \leq \frac{1}{2})$$

$$\begin{aligned} Z = X^2 + Y^2 &\quad \to X = \sqrt{W} \\ W = X^2 &\quad \to Y = \sqrt{Z - W} \end{aligned}$$

$$\rightarrow |\mathbf{J}| = \frac{1}{4\sqrt{Z-W}\sqrt{W}}$$

$$f_{Z,W} = \frac{3}{4\sqrt{z-w}} I_{(0,2)}(z) I_{(\frac{z}{2},z)}(w)$$
$$\rightarrow f_Z = \frac{3}{2}\sqrt{\frac{z}{2}} I_{(0,2)}(z)$$

$$P(X^2+Y^2 \leq \frac{1}{2}) = \frac{3}{2\sqrt{2}\int_0^{1/2}\sqrt{z}\mathtt{d}z} = \frac{1}{4}$$

(c)

$$P(X^2 \geq \frac{1}{2}|X+Y \leq 1) = \frac{P(X^2 \geq \frac{1}{2}, X+Y \leq 1)}{P(X+Y \leq 1)}$$

$$Z = X+Y \quad \rightarrow X = W$$
$$W = X \quad \rightarrow Y = Z-W$$

$$\rightarrow |\mathbf{J}| = 1$$

$$f_{Z,W} = 3wI_{(0,1)}(z)I_{(\frac{z}{2},z)}(w) + I_{(1,2)}(z)I_{(\frac{z}{2},1)}(w)$$
$$\rightarrow f_Z = \int_{\frac{z}{2}}^{z} 3w\mathtt{d}z$$

$$I_{(0,1)}(z) + \int_{\frac{z}{2}}^{1} 3w\mathtt{d}z \quad I_{(1,2)}(z)$$

$$= \frac{3}{2}(z^2 + \frac{z^2}{4}) \quad I_{(0,1)}(z) + \frac{3}{2}(z^2 + \frac{z^2}{4}) \quad I_{(1,2)}(z)$$

$$P(X+Y \leq 1) = \int_0^1 \frac{3}{2}(z^2 + \frac{z^2}{4})\mathtt{d}z = \frac{5}{8}$$

$$Z = X+Y \quad \rightarrow X = \sqrt{W}$$
$$W = X^2 \quad \rightarrow Y = Z - \sqrt{W}$$

$$\rightarrow |\mathbf{J}| = \frac{1}{2\sqrt{W}}$$

$$f_{Z,W} = \frac{3}{2} \quad 0 \leq z \leq 1, \frac{z^2}{4} \leq w \leq z^2 + 1 \leq z \leq 2, \frac{z^2}{4} \leq w \leq 1$$
$$\rightarrow P(X^2 \geq \frac{1}{2}, X+Y \leq 1) = \frac{3}{2}\int_{\frac{1}{2}}^{1}\int_{\sqrt{w}}^{1} \mathtt{d}z\mathtt{d}w = 0.069035$$
$$P(X^2 \geq \frac{1}{2}|X+Y \leq 1) = \frac{0.069035}{0.625}$$

(d)

$$P(X \geq \frac{1}{2} \cup Y \leq \frac{1}{2}) = P(X \geq \frac{1}{2}) + P(Y \leq \frac{1}{2}) - P(X \geq \frac{1}{2}, Y \leq \frac{1}{2})$$

$$P(X \geq \frac{1}{2}) = \int_{\frac{1}{2}}^{1} 3x^2 \mathtt{d}x = |frac78$$

$$P(Y \leq \frac{1}{2}) = \int_{0}^{\frac{1}{2}} 3x^2(1-y^2)\mathtt{d}y = \frac{11}{16}$$

$$P(X \geq \frac{1}{2}, Y \leq \frac{1}{2}) = \int_{\frac{1}{2}}^{1}\int_{0}^{\frac{1}{2}} 3x\mathtt{d}y\mathtt{d}x = \frac{9}{16}$$

$$P(X \geq \frac{1}{2} \cup Y \leq \frac{1}{2}) = 1$$

36. Sea

$$f_{X,Y} = c(\theta,\lambda)exp(\theta x + \lambda y) \quad I_{(x,y)}(a)$$

(a)

$$A = X^2 + Y^2 | x^2 + y^2 = 1$$

$$1 = c(\theta,\lambda)\int_0^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} exp(\theta x + \lambda y)\mathtt{d}y\mathtt{d}x + c(\theta,\lambda)\int_{-1}^0 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} exp(\theta x + \lambda y)\mathtt{d}y\mathtt{d}x$$

Usando polares

$$\begin{aligned} 1 =& \frac{c(\theta,\lambda)}{\lambda}\int_0^1 exp(\theta sin(\gamma) + \lambda cos(\gamma))cos\gamma\mathtt{d}\gamma \\ &- \frac{c(\theta,\lambda)}{\lambda}\int_0^1 exp(\theta sin\gamma - \lambda cos\gamma)cos\gamma\mathtt{d}\gamma \\ &+ \frac{c(\theta,\lambda)}{\lambda}\int_{-1}^0 exp(\theta sin\gamma + \lambda cos\gamma)cos\gamma\mathtt{d}\gamma \\ &- \frac{c(\theta,\lambda)}{\lambda}\int_{-1}^0 exp(\theta sin\gamma - \lambda cos\gamma)cos\gamma\mathtt{d}\gamma \end{aligned}$$

(b)

$$f_{X,Y} = c(\theta,\lambda)exp(\theta x + \lambda y)I_{(-1,1)}(x)I_{(-1,1)}(y)$$

$$1 = c(\theta,\lambda)\int_{-1}^1\int_{-1}^1 exp(\theta x + \lambda y)\mathtt{d}y\mathtt{d}x$$

$$\rightarrow c(\theta,\lambda) = \frac{\lambda\theta}{e^{\lambda+\theta} - e^{\lambda-\theta} + e^{\theta-\lambda} - e^{-\lambda-\theta}}$$

37. Sea $X_i|\theta \sim$ Ber(θ) $\quad \theta \sim$ Ber(α,β)

$$f(\theta, x_i) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\theta^{\alpha-1+x}(1-\theta)^{\beta-x} \quad I_{(0,1)}(\theta)I_{(0,1)}(x)$$

$$
\begin{aligned}
f_{X_i} &= \int_0^1 f_{x_i,\theta}\mathtt{d}\theta \\
&= \int_0^1 \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\theta^{\alpha-1+x}(1-\theta)^{\beta-x}\mathtt{d}\theta
\end{aligned}
$$

$$
\begin{aligned}
f(\theta|x_i) &= \frac{\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\theta^{\alpha-1+x}(1-\theta)^{\beta-x}}{\int_0^1 \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\theta^{\alpha-1+x}(1-\theta)^{\beta-x}\mathtt{d}\theta} \quad I_{(0,1)}(\theta) \\
&= \frac{\theta^{\alpha-1+x}(1-\theta)^{\beta-x}}{Beta(\alpha+x,\beta-x+1)} \quad I_{(0,1)}(\theta)
\end{aligned}
$$

38. Sea $X_i|\tau,\ \mu \sim N(\mu,\ \tau)$ $\quad\quad$ $\tau \sim$ Gamma $(\alpha,\ \beta)$

$$
\to f(\tau|x_i,\mu) = \frac{f(\tau,x_i)}{f(x_i)}
$$

$$
\to f(\tau,x_i) = \frac{\beta^\alpha}{\Gamma(\alpha)}\tau^{\alpha-1}e^{\beta x}\frac{\sqrt{\tau}}{\sqrt{2\pi}}e^{-\frac{\tau}{2}(x-\mu)^2} \quad I_{\Re}(\mu)I_{\Re^+}(\tau)I_{\Re}(x)
$$

$$
\begin{aligned}
f(x_i) &= \int_0^\infty \frac{\beta^\alpha}{\Gamma(\alpha)}\tau^{\alpha-1}e^{\beta x}\frac{\sqrt{\tau}}{\sqrt{2\pi}}e^{-\frac{\tau}{2}(x-\mu)^2}\mathtt{d}\tau \quad I_{\Re}(\mu)I_{\Re}(x) \\
&= \frac{\Gamma(\alpha+\frac{1}{2})}{-(\frac{(x-\mu)^2}{2})^{\alpha+1/2}} \quad I_{\Re}(\mu)I_{\Re}(x)
\end{aligned}
$$

$$
\begin{aligned}
f(\tau|x_i,\mu) &= \frac{\frac{\beta^\alpha}{\Gamma(\alpha)}\tau^{\alpha-1}e^{-\beta x}\frac{\sqrt{\tau}}{\sqrt{2\pi}}e^{-\frac{\tau}{2}(x-\mu)^2} \quad I_{\Re}(\mu)I_{\Re}(X)I_{\Re^+}(\tau)}{\int_0^\infty \frac{\beta^\alpha}{\Gamma(\alpha)}\tau^{\alpha-1}e^{-\beta x}\frac{\sqrt{\tau}}{\sqrt{2\pi}}e^{-\frac{\tau}{2}(x-\mu)^2}\mathtt{d}\tau \quad I_{\Re}(\mu)I_{\Re}(x)} \\
&= \frac{\tau^{\alpha-1}e^{-frac\tau 2(x-\mu)^2}}{\frac{\Gamma(\alpha+\frac{1}{2})}{-(\frac{(x-\mu)^2}{2})^{\alpha+\frac{1}{2}}}} \quad I_{\Re^+}(\tau)
\end{aligned}
$$

39. Sea X = Num de bola $\quad\quad$ Y = Num de Soles

(a)

$$
f_{XY} = \begin{cases}
x=1 \quad y=0 \quad \text{c.p. } \frac{2}{9} \\
x=1 \quad y=1 \quad \text{c.p. } \frac{2}{9} \\
x=2 \quad y=0 \quad \text{c.p. } \frac{1}{9} \\
x=2 \quad y=1 \quad \text{c.p. } \frac{1}{9} \\
x=2 \quad y=2 \quad \text{c.p. } \frac{1}{9} \\
x=3 \quad y=0 \quad \text{c.p. } \frac{1}{18} \\
x=3 \quad y=1 \quad \text{c.p. } \frac{1}{18} \\
x=3 \quad y=2 \quad \text{c.p. } \frac{1}{18} \\
x=3 \quad y=3 \quad \text{c.p. } \frac{1}{18}
\end{cases}
$$

(b)

$$f_X = \sum_{y=0}^{3} f_{XY} \quad I_{(1,2,3)}(x)$$

(c)

$$f_Y = \sum_{x=1}^{3} f_{XY} \quad I_{(0,1,2,3)}(y)$$

(d) Si existe correlacion lineal entre ellos, no son independientes.

(e)

$$20E(Y) = 20 * \frac{5}{6} = 16.6666, no \quad jugaria.$$

40. Sea X = Numero de veces que se obtuvieron i soles, i = 0, 1, 2, 3 y con

$$\sum X_i = 10$$

(a)

$$P(X_0 = 2, X_1 = 4, X_2 = 3, X_3 = 1) = \frac{10!}{2!3!4!}(\frac{1}{8})^2(\frac{2}{8})^3(\frac{3}{8})^4(\frac{1}{8})$$

(b)

$$P(X_2 < 2) = (\frac{5}{8})^{10} + 10(\frac{3}{8})(\frac{5}{8})^9$$

(c)

$$P(X_2^2 < 5) = P(X_2 < \sqrt{5}) = (\frac{5}{8})^{10} + 10(\frac{3}{8})(\frac{5}{8})^9$$

(d)

$$P(X_0 + X_1 = 8) = \sum_{x_0=0}^{8} \frac{8!}{x_0!(8-x_0)!}(\frac{1}{8})^{x_0}(\frac{3}{8})^{8-x_0}$$

(e)

$$P(X_2 < |X_1 = 5, X_0 = 3) = \frac{\sum_{x_2=0}^{1} \frac{(8+x_2)!}{x_2!5!3!}(\frac{1}{8})^2(\frac{3}{8})^5(\frac{3}{8})^{x_2}}{\frac{8!}{5!3!}(\frac{3}{8})^5(\frac{1}{8})^3}$$

41. Sean

$$u = \frac{y_1 - \mu_1}{\sigma_1}$$
$$v = \frac{y_2 - \mu_2}{\sigma_2}$$

$$f(y_2|y_1) = \frac{\frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} exp(-\frac{1}{1-\rho^2}(u^2 - 2\rho uv + v^2))}{\frac{1}{\sqrt{2\pi}\sigma_1} e^{-u^2/2}}$$
$$= \frac{1}{\sqrt{2\pi}\sigma_2\sqrt{1-\rho^2}} exp(-\frac{1}{2}(\frac{v-\rho u}{\sqrt{1-\rho^2}})^2)$$

$$\rightarrow f(y_2|y_1) = \frac{1}{\sqrt{2\pi}\sigma_2\sqrt{1-\rho^2}} exp(-\frac{1}{2}\frac{y_2 - (\mu_2 + \rho\frac{\sigma_2}{\sigma_1}(y_1 - \mu_1))}{\sigma_2\sqrt{1-\rho^2}})^2$$

$$f(y_2|y_1) \sim N(\mu_2 + \rho\frac{\sigma_2}{\sigma_1}(y_1 - \mu_1), \sigma_2^2(\sqrt{1-\rho^2})^2)$$

42. Sea

$$X \sim N(\mu, \Sigma)$$

Con

$$\mu = \begin{pmatrix} 3 \\ 3 \\ 0 \\ 0 \end{pmatrix} \quad \mathbf{\Sigma} = \begin{pmatrix} 2 & 0 & 2 & 0 \\ 0 & 1 & 1 & 0 \\ 2 & 1 & 5 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

(a)

$$|\Sigma| = 2, no \quad es \quad singular$$

(b)

$$\rho_{X_1,X_2} = \frac{2}{\sqrt{2}\sqrt{5}}$$

(c)

$$Y = CX + d$$

$$E(Y) = E(CX) + d = C\mu + d = \begin{pmatrix} 8 \\ 0 \\ 4 \end{pmatrix}$$

$$Vay(Y) = C\Sigma C^T = \begin{pmatrix} 13 & 9 & 11 \\ 9 & 13 & 11 \\ 11 & 11 & 11 \end{pmatrix}$$