Discussion on the paper: Principal applications of Bayesian methods in actuarial science. A perspective

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I would like to congratulate the editors for proposing a paper on Bayesian methods in actuarial sciences for discussion. The authors have presented a timely paper which deals with specific applications of the Bayesian paradigm to the solution of some actuarial problems and I hope that its publication will contribute to a further development of this area of research.

In order to have a general idea of the potential contribution of the Bayesian methods to the actuarial sciences it might be useful to start with two quotations. From the book by Bernardo and Smith (1994) we have: 'Bayesian Statistics offers a rationalist theory of personalistic beliefs in contexts of uncertainty, with the central aim of characterising how an individual should act in order to avoid certain kinds of undesirable behavioural inconsistencies. The theory establishes that expected utility maximisation provides the key to the ways in which beliefs should fit together in the light of changing evidence'. More recently, Lindley (2000) states: 'Inference is only of value if it can be used, so the extension to decision analysis, incorporating utility, is related to risk and to the use of statistics in science and law.' The point here is that Bayesian statistics is not merely a collection of methods for the analysis of data. It is a theory for making decisions under uncertainty where both components, probability and utility are equally important and a full Bayesian analysis includes data analysis, construction of probability models, assessment of the prior information and the utility function and, finally, making decisions.

In the financial world, making decisions under uncertainty has been recognized to be one of the most important activities. Foccardi and Jones (1998) have said: 'Risk measurement implies that there is a model of the market that, applied to data, mesures risk. But risk management is not limited to the more or less scientific process of measuring risk. Once measured, subjective judgement is used to evaluate and make decisions upon the measurement'.

Calculation of an appropriate premium and a reasonable level of reserves, the topics which are presented in the paper, are financial decision problems that can be formulated, within the Bayesian framework, in a very general setting. Particular solutions can be obtained corresponding to different choices for both,

the probabilistic representation of uncertainty and the specification of the utility function and even the dependence on these choices might be evaluated by comparing the different particular solutions. Many other actuarial problems can be treated in a similar fashion.

As for the experience rating problem discussed in the paper, it seems that for some time the proposed solutions had a Bayesian interpretation but they were not proper Bayesian solutions. It is interesting to notice that only recently the situation has been changing. Most of the contributions which explore the assessment of the prior probabilities and the utility function appeared in 1997 or latter and even the more general topic of robustness has been considered. It might be desirable to see in the future many contributions reporting posterior predictive distributions describing the observed experience and proposing utility functions which might be used to calculate premiums corresponding to different strategies and levels of protection.

Even though the authors clearly say that the paper does not aim at providing an extensive review of all applications of the Bayesian methodology, it is remarkable the absence of any comment in relation to the topic of Bayesian graduation. This subject has been explored for many years and some recent contributions have appeared which make use of MCMC and other simulation methods. The Bayesian approach to the construction of mortality tables was firstly considered in relation to the Whittaker graduation procedure which is not Bayesian but has a Bayesian interpretation. This relationship lead to further study of the conjugate multivariate normal model, see Kimeldorf and Jones (1967) and Hickman and Miller (1977), which apart from some variations of the Beta-Binomial analyisis was identified, for some time, as the Bayesian model for graduation. It is interesting to notice that these models do not incorporate any ordering restriction on the mortality rates and final product is the posterior distribution for the mortality rates and, more precisely, a Bayes estimate given by the posterior mean. Latter, in a paper by Broffit (1988) an exponential-gamma conjugate model is considered and different ways of impossing an increasing pattern among mortality rates were explored. More recently, Dellaportas and Smith (1993) have dealt with a closely related problem of making inferences in a class of of generalised linear models and proportional hazards models; Carlin (1992) discusses a Monte Carlo approach to Bayesian graduation and in particular, presents a simulation-based analysis of the model proposed by Broffit; Dellaportas, Smith and Stavropoulos (1999) concentrate on a Bayesian version of a model originally proposed by Heligman and Pollard (1980) which is intended to give a complete description of the mortality rates accross ages including the so-called accident hump. Mendoza, Madrigal and Gutiérrez-Peña (2001) have fitted a linear regression model to a set of transformed death rates and using predictive arguments have proposed a margin loaded table which is mandated for reserving purposes by the current Mexican Insurance regulations.

It is worth noticing that in the usual actuarial practice, the interest is focussed on the estimation of the true mortality rates. Hence, most of the the applications of the Bayesian paradigm to the problem of graduation are intended to produce these estimates which might be used as a mortality (or life)

table. In contrast, Mendoza, Madrigal and Gutiérrez-Peña (2001) derive the joint posterior distribution for the mortality rates to be observed in the future for a specific insured population and on that basis propose a strategy to select an appropriate mortality table.

Finally, I would like to congratulate the authors for this stimulating paper and as a last comment, and despite the already important number of valuable contributions, I would also have to stress that, in my opinion, the Bayesian approach is far from have being used in its full power to solve even the most well-known actuarial problems as for example, experience rating and graduation. Not only prior distributions and utility functions assessments have to be explored in further detail but also actuaries have to take advantage of the possibility of producing posterior predictive distributions describing the uncertainty that they will face in the future. An interesting reference in this direction is Cairns (2000). Other relevant references in the Bayesian literature which might have an impact in the future application of the Bayesian methods in the actuarial sciences are Bernardo and Muñoz (1994) where Bayesian analysis of demographical data is discussed, Schluter *et al.* (1997) where a Bayesian procedure is developed for ranking hazardous traffic accident locations, Denison *et al.* (1998) which deals with the problem of estimating a wide variety of curves using piecewise polynomials and Walker *et al.* (1999) where, among other results, a nonparametric approach is proposed to estimate predictive survival curves.

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