Closed Economy Solution to Ricardian Model

1. (a) Consumer's Maximization problem in country j:

Consumer's objective is to maximize utility by choosing consumption levels of the two goods, (X_c^j, Y_c^j), for given prices (p_x^j, p_y^j), subject to a budget constraint.

$$\max_{\{X_c^j, Y_c^j\}} U^j(X_c^j, Y_c^j) = X_c^{j\,\delta} Y_c^{j\,1-\delta}$$

$$\text{s.t.} \quad \underbrace{p_x^j \cdot X_c^j + p_y^j Y_c^j}_{\text{Expenditure}} = \underbrace{w_j \cdot \bar{L}^j}_{\text{Income}} \quad \} \to \text{Budget Constraint}$$

(b) Firms' maximization problem in country j:

Given the price of the good, ~~[illegible]~~ and the price of the factor(s), the firm's objective is to choose the level of inputs to maximize profits.
(The firms are price takers here → perfect competition)

Sector X:

$$\max_{\{L_x^j\}} \pi_x^j = p_x^j \cdot X_p^j - w_j L_x^j$$

$$\text{s.t.} \quad X_p^j = \alpha_j \cdot L_x^j$$

which is equivalent to

$$\max_{\{L_x^j\}} \pi_x^j = p_x^j \cdot \alpha_j L_x^j - w_j L_x^j$$

Similarly in Sector Y:

$$\max_{\{L_y^j\}} \pi_y^j = p_y^j \cdot \beta_j L_y^j - w_j L_y^j$$

Subscript 'p' refers to production

π – denotes profit

(c) Market clearing for goods & input(s):

$$X_c^j = X_p^j$$
$$Y_c^j = Y_p^j \quad \} \text{ Consumption} = \text{Production}$$

$$L_x^j + L_y^j = \bar{L}^j \quad \} \text{ Sum of labor force in X \& Y equals endowment of labor.}$$

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How to solve such a question!

The underlying idea is that this is a general equilibrium model, with some endogenous variables and some exogenously given parameters & variables.

<u>Endogenous Variables</u> : those that are chosen or determined within the model.

<u>Exogenous Variables</u> : those that are supplied from outside the model. These are taken as given.

Endogenous variables of the model are:

(i) consumption levels chosen by consumers $- \{X^j_c, Y^j_c\}$
(ii) Labor/employment chosen by firms $- \{L^j_x, L^j_y\}$
(iii) Goods' prices $- \{p^j_x, p^j_y\}$
(iv) Factor(s)/input(s) price(s) $- w_j$

Note that the output levels $\{X^j_p, Y^j_p\}$ are not listed above because once the firms choose $\{L^j_x, L^j_y\}$, then given the parameters $\{\alpha_j, \beta_j\}$ output levels are automatically determined.

Exogenous variables/parameters of the model are:

(i) Utility function parameter $- \delta$
(ii) Production function parameters $- \{\alpha_j, \beta_j\}$
(iii) Endowment of labor $- \bar{L}^j$

In <u>general equilibrium models</u>, we solve for (or determine) the endogenous variables, given some exogenous variables/parameters.

In other models, termed as <u>partial equilibrium models</u>, some or all prices (i.e goods & factor prices) are also taken as exogenous variables.

So, the objective is to solve for 7 endogenous variables:

$$\{ X_c^j, Y_c^j, L_x^j, L_y^j, p_x^j, p_y^j, w_j \}$$

To uniquely determine these variables we will need 7 equations. We get these equations by solving the consumer's maximization problem, the firms' profit maximization problem & using the market clearing conditions. An important condition is that the 7 equations must be independent. We will come to this later.

Lets start....

Consumer's maximization problem:

$$\underset{\{X_c^j, Y_c^j\}}{\text{Max}} \quad u^j(X_c^j, Y_c^j) = X_c^{j\,\delta} Y_c^{j\,1-\delta}$$

$$\text{s.t} \quad p_x^j . X_c^j + p_y^j . Y_c^j = w_j \} \rightarrow (\bar{L}^j = 1) \; (\Rightarrow w_j \bar{L}^j = w_j)$$

Then the Lagrangian is given by

$$\mathcal{L} = X_c^{j\,\delta} Y_c^{j\,1-\delta} - \lambda^j [p_x^j . X_c^j + p_y^j . Y_c^j - w_j]$$

$\lambda^j \rightarrow$ Lagrange Multiplier

Partially differentiate '$\mathcal{L}$' with respect to X_c^j, Y_c^j & λ^j and equate each resulting expression to zero. These equations are often called first-order conditions.

$$\frac{\partial \mathcal{L}}{\partial X_c^j} = 0 \Rightarrow \delta X_c^{j\,\delta-1} . Y_c^{j\,1-\delta} - \lambda^j p_x^j = 0 \quad — ①$$

$$\frac{\partial \mathcal{L}}{\partial Y_c^j} = 0 \Rightarrow (1-\delta) X_c^{j\,\delta} Y_c^{j\,-\delta} - \lambda^j p_y^j = 0 \quad — ②$$

$$\frac{\partial \mathcal{L}}{\partial \lambda^j} = 0 \Rightarrow w_j - p_x^j . X_c^j - p_y^j . Y_c^j = 0 \quad — ③$$

Dividing ① by ② gives

$$\frac{\delta}{(1-\delta)} . \frac{Y_c^j}{X_c^j} = \frac{p_x^j}{p_y^j} \quad — ④$$

The firms' profit maximization problem:

Sector X: $\max\limits_{\{L_x^j\}} \pi_x^j = p_x^j \cdot \alpha_j \cdot L_x^j - w_j L_x^j$

Partially differentiating the expression for profits with respect to L_x^j

$$\frac{\partial \pi_x^j}{\partial L_x^j} = 0 \Rightarrow p_x^j \alpha_j - w_j = 0 \qquad \text{— (5)}$$

Sector Y: $\max\limits_{\{L_y^j\}} \pi_y^j = p_y^j \beta_j L_y^j - w_j L_y^j$

$$\frac{\partial \pi_y^j}{\partial L_y^j} = 0 \Rightarrow p_y^j \cdot \beta_j - w_j = 0 \qquad \text{— (6)}$$

Market Clearing:

$$X_c^j = X_p^j = X^j \qquad \text{— (7)}$$

$$Y_c^j = Y_p^j = Y^j \qquad \text{— (8)}$$

$$L_x^j + L_y^j = 1 \qquad \text{— (9)}$$

Putting things together, we get 7 equations { (3), (4), (5), (6), (7), (8) & (9) }. However, as I had said earlier, these equations have to be independent. Equation (7) & (8) are not independent because of the Walras' Law. Walras' Law states that for 'n' goods, if the demand is equal to supply for (n - 1) goods, then demand equals supply for the 'nth' good as well. This means that if markets clear for the first '(n-1)' goods, then market clears for the 'nth' good automatically.

As a result, we can use only one of these equations, implying we have only 6 independent equations. Thus, we are short of one equation.

So, we have 7 endogenous variables and 6 equations. The way solve this problem is by assuming that the wage wage in the country is 1. What this means is that the price of the goods is expressed in terms of wage paid to labor. Therefore, these are relative prices (relative to wage).

So $w_j^i = 1$, for $i = \{h, f\}$

2.) From, (5) & (6)

$$p_x^j \alpha_j = w_j \Rightarrow p_x^j = 1/\alpha_j$$

$$p_y^j \beta_j = w_j \Rightarrow p_y^j = 1/\beta_j$$

Taking a ratio implies

$$\boxed{\frac{p_x^j}{p_y^j} = \frac{\beta_j}{\alpha_j}} \quad \text{—} \quad (10)$$

$$\therefore \frac{p_x^h}{p_y^h} = \frac{20}{20} = 1 \quad \& \quad \frac{p_x^f}{p_y^f} = \frac{10}{30} = 1/3$$

3.) The wage ratio is $\frac{w_h}{w_f} = 1$ (due to our normalization)

4.) From equation (4) $\frac{Y_c^j}{X_c^j} = \frac{(1-\delta)}{\delta} \frac{p_x^j}{p_y^j} = \frac{(1-\delta)}{\delta} \cdot \frac{\beta_j}{\alpha_j}$

$$\Rightarrow Y_c^j = \frac{(1-\delta)}{\delta} \frac{\beta_j}{\alpha_j} \cdot X_c^j \quad \text{—} \quad (11)$$

Since $Y_c^j = Y^j = \beta_j L_y^j \Rightarrow L_y^j = Y^j/\beta_j$

$X_c^j = X^j = \alpha_j L_x^j \Rightarrow L_x^j = X^j/\alpha_j$

Substitute for L_x^j & L_y^j the labor market clearing condition (Eq (9))

$$\frac{X^j}{\alpha_j} + \frac{Y^j}{\beta_j} = 1$$

Substituting for Y^j from (11)

$$\Rightarrow \frac{X^j}{\alpha_j} + \frac{(1-\delta)}{\delta} \frac{\cancel{\beta_j}}{\alpha_j} \frac{X^j}{\cancel{\beta_j}} = 1$$

$$\Rightarrow \frac{X^j}{\alpha_j}\left[1 + \frac{(1-\delta)}{\delta}\right] = 1$$

$$\therefore \boxed{X^j = \delta \alpha_j} \quad — \quad (12)$$

Then from (11),

$$Y^j = \frac{(1-\delta)}{\cancel{\delta}} \cdot \frac{\beta_j}{\cancel{\alpha_j}} \cdot \cancel{\delta}\cancel{\alpha_j}$$

$$\therefore \boxed{Y^j = (1-\delta)\beta_j} \quad — \quad (13)$$

Utility level in country j is

$$U^j = X^{j\,\delta} Y^{j\,1-\delta}$$

$$= (\delta\alpha_j)^{\delta}((1-\delta)\beta_j)^{1-\delta}$$

$$\boxed{U^j = \delta^{\delta}(1-\delta)^{1-\delta} \cdot \alpha_j^{\delta} \beta_j^{1-\delta}} \quad — \quad (14)$$

$\therefore U^h = 0.5^{0.5} \times 0.5^{0.5} \times (20)^{0.5} \times (20)^{0.5}$ & $U^f = 0.5^{0.5} \times 0.5^{0.5} \times (30)^{0.5} \times (10)^{0.5}$

$U^h = 0.5 \times 20$. $U^f = 0.5 \times (30)^{0.5} \times (10)^{0.5}$

$\Rightarrow U^h = 10$ $\Rightarrow U^f = 8.66$