FINAL - FALL, 2014

ECO-13101 Economia Internacional I (International Trade Theory)

December 12, 2014

Instructions: *Write your name and clave on the first page of your answer booklet/sheets and on the question booklet/sheets. Number all the sheets carefully, and staple them with the question sheets. You have 2 hours and 45 minutes to finish the exam. All questions have to be answered. All the best!*

Question 1 (60 points)

Consider trade between N ($N \geq 2$) countries. A representative consumer in country j maximizes

$$U_j = \sum_{i=1}^{N} n_i \left(C_{ij}\right)^{\frac{\sigma-1}{\sigma}} \ , \ \sigma > 1 \ ,$$

where C_{ij} is the consumption of any product sent from country i to country j, and σ is the elasticity of substitution between products. Due to monopolistic competition each firm in each country produces a unique product which is sold domestically and is also exported. We are assuming that all products exported by country i sell for the same price p_{ij} in country j. n_i is the number of products produced (and exported) by country i. Labor is the only factor of production. Endowment of labor in country j is $\overline{L}_j$, and wage is w_j.

1. **(10 points)** Setup and solve the the consumer's utility maximization exercise to show that

$$C_{ij} = \frac{w_j \overline{L}_j}{P_j} \left(\frac{p_{ij}}{P_j}\right)^{-\sigma}$$

   where $P_j = \left(\sum_i^N n_i \left(p_{ij}\right)^{1-\sigma}\right)^{1/(1-\sigma)}$ is price index of all products consumed in country j.

2. **(15 points)** Suppose, every product produced in country i requires $L_i = \alpha + \beta_i . X_i$ amount of labor to produce output X_i. α is fixed labor input (fixed cost), β_i is a constant marginal labor input. This implies that every product produced by country i is sold at the same price p_i. Setup and solve the firm's profit maximization problem to show that

$$p_i = \frac{\sigma}{\sigma - 1} \beta_i w_i \ .$$

   Show that price is greater than marginal cost. What is the mark-up?

3. **(10 points)** Since firms are monopolistically competitive they earn zero profit. Use this to show that the number of products produced by country i is given by

$$n_i = \frac{\overline{L}_i}{\sigma\alpha} \quad .$$

4. **(25 points)** Suppose transporting any product from country i to country j incurs a per unit transportation cost of τ_{ij}. Then the price of a product in country j, imported from country i, is $p_{ij} = \tau_{ij} p_i$, where p_i is the price charged by firms of country i (as derived in (2)).

   (a) **(10 points)** Substitute this price of imported goods in the expression for C_{ij} (derived in (1)) and show that the total expenditure by country j on goods imported from country i is given by

   $$T_{ij} = n_i Y_j \left(\frac{\tau_{ij} p_i}{P_j}\right)^{1-\sigma} \quad ,$$

   where $Y_j = w_j \overline{L}_j$.

   (b) **(15 points)** Use the expressions for n_i and p_i to derive the following gravity equation

   $$T_{ij} = \frac{\beta_i}{\alpha(\sigma-1)} \left(\frac{Y_i Y_j}{p_i^{\sigma}}\right) \left(\frac{\tau_{ij}}{P_j}\right)^{1-\sigma} \quad ,$$

   where $Y_i = w_i \overline{L}_i$. Explain the effect of an increase in τ_{ij} on T_{ij}. How does this depend on the value of σ? Give intuition for your explanation.

Question 2 (40 points)

Consider the Melitz model of trade with monopolistically competitive firms and two identical countries. Each firm produces a unique variety, which it can sell domestically as well as export it. The price (p), revenue (r), and profit (π) of a firm with labor productivity φ in the domestic (d) and export (x) market are given by

$$p(\varphi) = \begin{cases} p_d(\varphi) = \frac{\sigma}{\sigma-1} \cdot \frac{1}{\varphi}, & \text{domestic market} \\ p_x(\varphi) = \frac{\sigma}{\sigma-1} \cdot \frac{\tau}{\varphi}, & \text{export market} \end{cases}$$

$$r(\varphi) = \begin{cases} r_d(\varphi) = \left(\frac{\sigma}{\sigma-1} \cdot \varphi P\right)^{\sigma-1} R, & \text{domestic market} \\ r_d(\varphi) + r_x(\varphi), & \text{both markets} \end{cases}$$

where $r_x(\varphi) = \left(\frac{\sigma}{\sigma-1} \cdot \frac{\varphi}{\tau} P\right)^{\sigma-1} R$

$$\Pi(\varphi) = \begin{cases} \Pi_d(\varphi), & \text{domestic market} \\ \Pi_d(\varphi) + \Pi_x(\varphi), & \text{both markets} \end{cases}$$

where $\Pi_d(\varphi) = \left(\frac{(\sigma-1)}{\sigma}\varphi P\right)^{\sigma-1}\frac{R}{\sigma} - f$ and $\Pi_x(\varphi) = \left(\frac{(\sigma-1)}{\sigma}\frac{\varphi}{\tau}P\right)^{\sigma-1}\frac{R}{\sigma} - f_x$. P is the price index, R is the total expenidture on all varieities consumed, $f$and f_x are the fixed costs of production for domestic and export market, respectively, $\tau > 1$ is the iceberg trade cost, and $\sigma > 1$ is the elasticity of substitution. Let the endowment of labor be L. Let $\underline{\varphi}$ and $\underline{\varphi}_x$ denote the cut-off productivity to enter the domestic and the export market, respectively, and the average productivity of operating firms by $\tilde{\varphi}$. Wages are normalized to one, and we assuume that there is free antry and exit of firms. Let f_e denote the fixed cost of entering the industry, $G(\varphi)$ be the distribution from which φ is drawn, and let $g(\varphi)$ be the probability density function.

1. **(20 points)** Denote the equilibrium cut-off productivities by $\underline{\varphi}^*$, $\underline{\varphi}_x^*$, and the equilibrium average profit by $\Pi(\tilde{\varphi}^*)$. Show that the open economy equilibrium is the solution to the ZCP and FE conditions which are given by

$$\begin{array}{lrcl} \text{(ZCP)} & \Pi(\tilde{\varphi}) & = & \kappa(\underline{\varphi}^*)f + prob_x \kappa(\underline{\varphi}_x^*)f_x \quad , \\ \text{(FE)} & \Pi(\tilde{\varphi}) & = & \dfrac{f_e}{1-G(\underline{\varphi}^*)} \quad , \end{array}$$

   where

$$\begin{aligned} \kappa(\underline{\varphi}^*) &= \left[\left(\frac{\tilde{\varphi}^*(\underline{\varphi}^*)}{\underline{\varphi}^*}\right)^{\sigma-1} - 1\right] \quad , \\ \kappa(\underline{\varphi}_x^*) &= \left[\left(\frac{\tilde{\varphi}_x^*(\underline{\varphi}_x^*)}{\underline{\varphi}_x^*}\right)^{\sigma-1} - 1\right] \quad , \\ \underline{\varphi}_x^* &= \tau\left(\frac{f_x}{f}\right)^{1/(\sigma-1)}\underline{\varphi}^* \quad , \\ prob_x &= \frac{1-G(\underline{\varphi}_x^*)}{1-G(\underline{\varphi}^*)} \quad . \end{aligned}$$

   Depict the equilibrium graphically.

2. **(20 points)** Show that $\partial\underline{\varphi}^*/\partial\tau < 0$ and $\partial\underline{\varphi}_x^*/\partial\tau > 0$. Interpret this result graphically, and explain the effect on selection of firms in the domestic and the export market. (*Hints: (i) Combine the ZCP and FE conditions, (ii) differentiate w.r.t.* τ, *(iii) use the link between* $\underline{\varphi}^*$ *and* $\underline{\varphi}_x^*$, *and (iv) use the following result:* $\frac{\partial\kappa(\varphi)}{\partial\varphi} = \frac{\kappa(\varphi)g(\varphi)}{1-G(\varphi)} - \frac{(\sigma-1)[\kappa(\varphi)+1]}{\varphi} \quad \forall\ \varphi$)