# Simple Melitz Model

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Melitz(2003) adds heterogeneity in productivity to the Krugman model.
↳ Firms differ in their marginal productivity of labor.

Demand side: $U = (\int_{w\in\Omega} q_w^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}}$

where

$\omega$ - represents a unique variety

$\Omega$ - set of all varieties supplied

Budget constraint is simply

$$\underbrace{\int_{w\in\Omega} p_w q_w}_{R} = wL = L$$

where R is the agregate expenditure. (normalize $w = 1$)

Solving the utility maximization problem gives the demand for a variety w to be

$$q_w = \left(\frac{p_\omega}{P}\right)^{-\sigma} \frac{R}{P} \quad \left(\text{or } \frac{p_\omega^{-\sigma} L}{\sum_{\nu\in\Omega} p_\nu^{1-\sigma}}\right)$$

where $P = \left(\int_{\nu\in\Omega} p_\nu^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$ is the price index of all varieties.

Supply side: each variety $\omega$ is produced by a unique firm. But unlike the Krugman model, firms are heterogeneous in the labor productivity with which they produce a variety.

↳ A firm producing variety $\omega$ needs the following units of labor

$$l(q, \varphi_\omega) = f + \frac{q_\omega}{\varphi_\omega}$$

where

$q_\omega$ - output variety of $\omega$

f - fixed cost of production

$\varphi_\omega$ - productivity of labor for the firm

$\left(\frac{1}{\varphi_\omega}\right) \to$ marginal cost of production

Solving the firm's profit maximization problem, the price charged by producer of variety $\omega$ is

$$p_\omega = \frac{\sigma}{(\sigma-1)\varphi_\omega} \qquad \text{(impose demand=supply)}$$

The revenue of the firm is

$$r(\varphi_\omega) = \left(\frac{(\sigma-1)}{\sigma}\varphi_\omega \cdot P\right)^{\sigma-1} \cdot R$$

The profit of the firm is

$$\Pi(\varphi_\omega) = \left(\frac{(\sigma-1)}{\sigma}\varphi_\omega \cdot P\right)^{\sigma-1} \cdot \frac{R}{\sigma} - f$$

<u>Some implications so far</u>: for two firms such that $\left(\frac{p_\omega}{p_{\omega'}}\right)^{-\sigma}$

(i) $\frac{q_\omega}{q_{\omega'}} = \left(\frac{p_\omega}{p_{\omega'}}\right)^{-\sigma} = \left(\frac{\varphi_\omega}{\varphi_{\omega'}}\right)^{-\sigma}$ $\}$ Since $\sigma > 1 \Rightarrow q_\omega > q_{\omega'}$ more productive firms sell more output.

(ii) $\frac{p_\omega}{p_{\omega'}} = \frac{\varphi_\omega}{\varphi_{\omega'}}$ $\}$ More productive firms charge lower prices

(iii) $\frac{r(\varphi_\omega)}{r(\varphi_{\omega'})} = \left(\frac{\varphi_\omega}{\varphi_{\omega'}}\right)^{\sigma-1}$ $\}$ More productive firms earn more revenue.

(iv) $\frac{\partial\pi(\varphi_\omega)}{\partial\varphi_\omega} = (\sigma-1)\varphi_\omega^{\sigma-2} \cdot \left(\frac{\sigma-1}{\sigma} \cdot P\right)^{\sigma-1} \cdot \frac{R}{\sigma} > 0 \Rightarrow$ More productive firms earn higher profits.

<u>Cutoff Productivity</u>:

- Since profits are increasing in productivity $\left(\frac{\partial\pi(\varphi_\omega)}{\partial\varphi_\omega} > 0\right)$, a firm will choose to supply if

$$\pi(\varphi_\omega) > 0$$

Thus, there exists a unique $\underline{\varphi}$ s.t.

$$\begin{array}{llcll} & \pi(\underline{\varphi}) & = & 0 & \\ \& & \pi(\varphi) & > & 0 & \forall\varphi > \underline{\varphi} \\ \& & \pi(\varphi) & < & 0 & \forall\varphi < \underline{\varphi}\} \text{ These firms do not produce} \end{array}$$

- $\varphi \sim g(\varphi)$ with support $(0, \infty)$
- firms that draw a $\varphi \geq \underline{\varphi}$ will supply the market and this therefore determines the set of varieties supplied to the market.
- The distribution of productivities of firms operating in the market is given by

  $$\mu(\varphi) = \left\{ \begin{array}{ll} \frac{g(\varphi)}{1-G(\underline{\varphi})} & \text{if } \varphi \geq \underline{\varphi} \\ 0 & \text{otherwise} \end{array} \right\rangle \mu \text{ is conditional distribution of } g \text{ on } [\underline{\varphi}, \infty)$$

  where $1 - G(\underline{\varphi})$ is the probability of drawing $\varphi \geq \underline{\varphi}$.

Aggregation: Define aggregate productivity level

$$\widetilde{\varphi}(\varphi) = \left( \int_0^\infty \varphi^{\sigma-1} \mu(\varphi) d\varphi \right)^{\frac{1}{\sigma-1}} = \left( \frac{1}{1 - G(\underline{\varphi})} \int_{\underline{\varphi}}^\infty \varphi^{\sigma-1} g(\varphi) d\varphi \right)^{\frac{1}{\sigma-1}}$$

- Let M be the mass of firms with $\varphi \geq \underline{\varphi}$. Then

  $$P = \left[ \int_0^\infty p(\varphi)^{1-\sigma} \cdot M\mu(\varphi) d\varphi \right]^{\frac{1}{1-\sigma}}$$

  since $p(\varphi) = \frac{\sigma}{(\sigma-1)\varphi_\omega}$

$$\begin{aligned} \Rightarrow \quad P &= \left[ \int_0^\infty \left( \frac{\sigma}{\sigma-1} \cdot \frac{1}{\varphi} \right)^{1-\sigma} \cdot M\mu(\varphi) d\varphi \right]^{\frac{1}{1-\sigma}} \\ P &= M^{\frac{1}{1-\sigma}} \frac{\sigma}{\sigma-1} \left[ \int_0^\infty \varphi^{\sigma-1} \cdot \mu(\varphi) d\varphi \right]^{\frac{1}{1-\sigma}} \\ P &= M^{\frac{1}{1-\sigma}} \frac{\sigma}{\sigma-1} \frac{1}{\left[ \int_0^\infty \varphi^{\sigma-1} \cdot \mu(\varphi) d\varphi \right]^{\frac{1}{\sigma-1}}} \\ \therefore \quad P &= M^{\frac{1}{1-\sigma}} \frac{\sigma}{(\sigma-1)\tilde{\varphi}} = M^{\frac{1}{1-\sigma}} \cdot p(\tilde{\varphi}) \end{aligned}$$

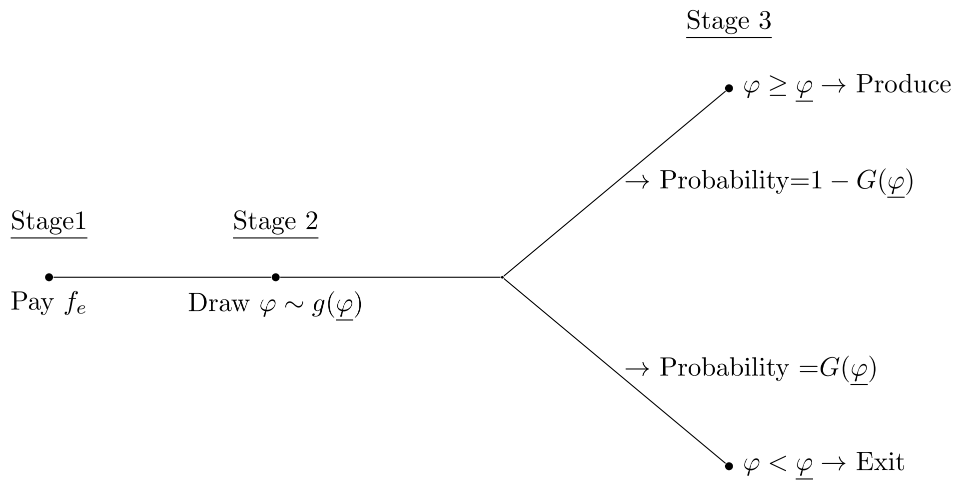
Similarly it can be shown that

$$\begin{aligned} R &= M \cdot r(\tilde{\varphi}) \\ \pi &= M \cdot \pi(\tilde{\varphi}) \end{aligned}$$

Thus, an industry comprised of $M$ firms with any distribution of productivity levels $\mu(\varphi)$ that yields the same average productivity $\tilde{\varphi}$ will also induce the same aggregate outcome as an industry with M representative firms sharing the same $\varphi = \tilde{\varphi}$.

Equilibrium:

- There is a large (unbounded) pool of prospective entrants into the industry.
- Prior to entry firms are identical.
- To enter, a firm must make an initial investment- fixed cost - $f_e$ -, which is sunk after entry. This is measured in units of labor.
- Then the firm draws $\varphi \sim g(\varphi)$
  ↳ The sunk cost prevents firms drawing $\varphi$ repeatedly till they receive a profitable draw and hence prevent $M$ from growing unboundedly.
  ↳ One way of thinking about $f_e$ is as development cost of a new variety.




- Thus, firms, prior to entry, think about their expected profits.
- Since all incumbent firms, except the cutoff, earn positive profit, the average profit level must be positive, which is what attracts entry.
- Since the average productivity level $\tilde{\varphi}$ is determined by the cutoff productivity $\underline{\varphi}$, so are average revenue and average profit. (conditional on entry)

  Since $r(\varphi) = \left(\frac{(\sigma-1)}{\sigma}\varphi P\right)^{\sigma-1} \cdot R$

$$\begin{array}{rrcl} \Rightarrow & \frac{r(\tilde{\varphi})}{r(\underline{\varphi})} & = & \left(\frac{\tilde{\varphi}(\underline{\varphi})}{\underline{\varphi}}\right)^{\sigma-1} \\ \Rightarrow & r(\tilde{\varphi}) & = & \left(\frac{\tilde{\varphi}(\underline{\varphi})}{\underline{\varphi}}\right)^{\sigma-1} \cdot r(\underline{\varphi}) \\ Then & \Pi(\tilde{\varphi}) & = & \frac{r(\tilde{\varphi}))}{\sigma} - f \\ & & = & \left(\frac{\tilde{\varphi}(\underline{\varphi})}{\underline{\varphi}}\right)^{\sigma-1} \cdot \frac{r(\underline{\varphi})}{\sigma} - f \end{array}$$

- For the cutoff firm

$$
\begin{array}{llcl}
 & \Pi(\underline{\varphi}) & = & 0 \\
\Rightarrow & \frac{r(\underline{\varphi})}{\sigma} - f & = & 0 \\
\Rightarrow & r(\underline{\varphi}) & = & \sigma f \\
\therefore & r(\tilde{\varphi}) & = & \left(\frac{\tilde{\varphi}(\underline{\varphi})}{\underline{\varphi}}\right)^{\sigma-1} \cdot \sigma f \\
\& & \Pi(\tilde{\varphi}) & = & \left(\frac{\tilde{\varphi}(\underline{\varphi})}{\underline{\varphi}}\right)^{\sigma-1} \cdot \frac{\sigma f}{\sigma} - f = \underbrace{\left[\left(\frac{\tilde{\varphi}(\underline{\varphi})}{\underline{\varphi}}\right)^{\sigma-1} - 1\right]}_{\kappa(\underline{\varphi})} f
\end{array}
$$

- What is the expected value from entry.

$$
V^e = \underbrace{(1 - G(\underline{\varphi})}_{\substack{\text{Probability} \\ \text{of entry}}} \cdot \underbrace{\Pi(\tilde{\varphi})}_{\substack{\text{Ex ante} \\ \text{profits} \\ \text{conditional} \\ \text{on entry}}} - f_e
$$

- Free entry (as in Krugman) $\Rightarrow V^e = 0 \rightarrow$ True only for a stationary equilibrium.
( Along a transition $V^e < 0$. Thus, the correct condition is $V^e \leq 0$.)

- Equilibrium is a $\left(\underline{\varphi}, \Pi(\tilde{\varphi})\right)$ and mass $M$ of firms such that

$$
\begin{array}{llcll}
(ZCP) & \Pi(\tilde{\varphi}) & = & \left[\left(\frac{\tilde{\varphi}(\underline{\varphi})}{\underline{\varphi}}\right)^{\sigma-1} - 1\right] f = \kappa(\underline{\varphi}) \cdot f & (I) \\
(FE) & V^e & = & 0 \Rightarrow \Pi(\tilde{\varphi}) = \frac{f_e}{1-G(\underline{\varphi})} & (II) \\
(LMC) & L & = & R & (III)
\end{array}
$$

The $LMC$ (labor market clearing condition) implies

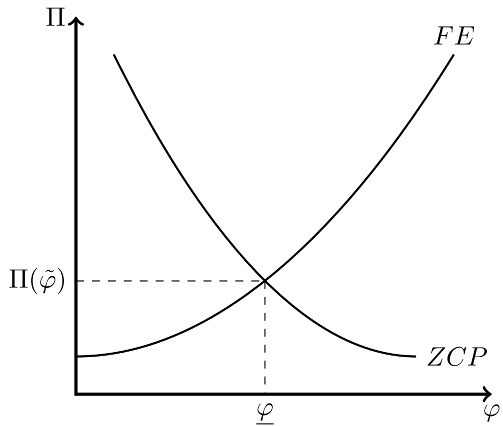
$$
\begin{array}{lccl}
 & L & = & Mr(\tilde{\varphi}) \\
\Rightarrow & L & = & M\left(\frac{\tilde{\varphi}(\underline{\varphi})}{\underline{\varphi}}\right)^{\sigma-1} \cdot \sigma f
\end{array}
$$

Since $\Pi(\tilde{\varphi}) = \left(\frac{\tilde{\varphi}(\underline{\varphi})}{\underline{\varphi}}\right)^{\sigma-1} \cdot \frac{\sigma f}{\sigma} - f$

$$
\begin{array}{lrcl}
\Rightarrow & \left(\frac{\tilde{\varphi}(\underline{\varphi})}{\underline{\varphi}}\right)^{\sigma-1} \cdot \sigma f & = & \sigma(\Pi(\tilde{\varphi}) + f) \\
\therefore & L & = & M \cdot \sigma(\Pi(\tilde{\varphi}) + f) \\
\therefore & M & = & \frac{L}{\sigma(\Pi(\tilde{\varphi})+f)}
\end{array}
$$

- $ZCP$ and $FE$ give us a solution for $\Pi(\tilde{\varphi})$ and $\underline{\varphi}$ which is then plugged into $(III)$ to get the $M$ in equilibrium.

Given $\Pi(\tilde{\varphi})$, $\underline{\varphi}$, and $M$ we can cover aggregate output, prices and production of firms.
↳ For a unique solution to exist, $\frac{\varphi g(\varphi)}{G(\varphi)}$ must be increasing, which is satisfied for most distributions.




↳ FE is increasing in $\underline{\varphi}$ because as cutoff productivity increases $\Rightarrow$ fewer firms will be able to enter $\Rightarrow$ those who enter will earn higher profits $\Rightarrow$ average profit conditional on entry is going to be higher.
↳ $ZCP$ is decreasing in $\underline{\varphi}$ as long as $\frac{g(\varphi)\varphi}{1-G(\varphi)}$ is increasing infinitely on $(0, +\infty)$

<u>Welfare</u>:
Here welfare is simply the real wage $\frac{w}{P}$.
Since $w = 1$

$$\Rightarrow \text{ welfare } = P^{-1} = \frac{M^{\frac{1}{\sigma-1}}}{p(\tilde{\varphi})}$$

Thus welfare is increasing in $M \to$ number of varieties. And it is decreasing in the average price.

<u>Open Economy Case</u>:

- If there where two such cases and there where no costs of trade, then in the trade equilibrium thing would work as if the economy's size had doubled and it were closed.

  - But this would not affect any of the firm level outcomes - prices, quantity , revenues and profits.
  - The same number of firms in each country produce at the same output level and earn the same profits as they did under autarky.

- In the absence of any trade costs firm heterogeneity does not impact the effect of trade.

- However, there is enough evidence that there are not only per unit trade costs but also fixed costs of exporting
  - information foreign buyers, learning about new markets, regulatory costs, distribution networks

Setup of the Trade Model (2 countries)

- A firm who wishes to export must make an initial fixed investment ($f_x$), but this investment decision occurs after the firm knows its productivity $\varphi$.
- Furthermore, there is a per-unit trade cost $\tau$.
  - Iceberg assumption $\Rightarrow \tau > 1$ units of goods must be shipped for 1 unit to arrive at the destination.
- Countries are identical - all countries wage - $w = 1$
  - Ensure factor price equalization and hence abstracts from relative wage differences driven firm selection and aggregate productivity.
  - All aggregate variables $(L, R, P)$ are equal across countries.

Exporting firms:

- Since the firm first draws its productivity and the decides whether or not it exports, all exporting firms must sell domestically (but converse is not true)

Pricing :

$$p(\varphi) = \begin{cases} \frac{\sigma}{\sigma-1} \cdot \frac{1}{\varphi}, & \text{domestic market} \\ \frac{\sigma}{\sigma-1} \cdot \frac{\tau}{\varphi}, & \text{foreign market} \end{cases}$$

Thus $p_x(\varphi) = \frac{\sigma}{\sigma-1} \cdot \frac{\tau}{\varphi} = \tau p_d$ where $p_d(\varphi) = \frac{\sigma}{\sigma-1} \cdot \frac{1}{\varphi}$

Revenue :

$$r(\varphi) = \begin{cases} r_d(\varphi) = \left(\frac{\sigma}{\sigma-1} \cdot \varphi P\right)^{\sigma-1} R, & \text{domestic market} \\ r_d(\varphi) + r_x(\varphi), & \text{both markets} \end{cases}$$

where $r_x(\varphi) = \left(\frac{\sigma}{\sigma-1} \cdot \frac{\varphi}{\tau} P\right)^{\sigma-1} R = \sigma^{1-\sigma} \cdot r_d(\varphi)$.

Profits :

$$\Pi(\varphi) = \begin{cases} \Pi_d(\varphi), & \text{domestic market} \\ \Pi_d(\varphi) + \Pi_x(\varphi), & \text{both markets} \end{cases}$$

where

$$\begin{aligned} \Pi_d(\varphi) &= \left(\frac{(\sigma-1)}{\sigma}\varphi P\right)^{\sigma-1} \cdot \frac{R}{\sigma} - f \\ \Pi_x(\varphi) &= \left(\frac{(\sigma-1)}{\sigma}\frac{\varphi}{\tau} P\right)^{\sigma-1} \cdot \frac{R}{\sigma} - f_x \end{aligned}$$

Cutoff Productivity for Exporting:

- All firms with a $\varphi$ s.t. $\Pi_x(\varphi) \geq 0$ will export.

$$\begin{aligned}
\therefore \qquad \Pi_x(\underline{\varphi}_x) &= 0 \\
\Rightarrow \quad \left(\frac{(\sigma-1)}{\sigma}\frac{\underline{\varphi}_x}{\tau}P\right)^{\sigma-1} \cdot \frac{R}{\sigma} - f_x &= 0
\end{aligned}$$

Remember for $\varphi = \underline{\varphi}$

$$\begin{aligned}
\Pi_d(\underline{\varphi}) &= 0 \\
\Rightarrow \quad \left(\frac{(\sigma-1)}{\sigma}\underline{\varphi}P\right)^{\sigma-1} \cdot \frac{R}{\sigma} - f &= 0 \\
\Rightarrow \quad \left(\frac{(\sigma-1)}{\sigma}P\right)^{\sigma-1} \cdot \frac{R}{\sigma} &= \frac{f}{(\underline{\varphi})^{\sigma-1}}.
\end{aligned}$$

$$\begin{aligned}
\therefore \quad \left(\frac{\underline{\varphi}_x}{\underline{\varphi}\tau}\right)^{\sigma-1} \cdot f &= f_x \\
\Rightarrow \quad \underline{\varphi}_x &= \tau\left(\frac{f_x}{f}\right)^{\frac{1}{\sigma-1}} \cdot \underline{\varphi}
\end{aligned}$$

We will assume that $\tau\left(\frac{f_x}{f}\right)^{\frac{1}{\sigma-1}} > 1$ so that

$$\underline{\varphi}_x > \underline{\varphi}$$

Thus we get the following partitions:

(i) $\varphi < \underline{\varphi}$ $\rightarrow$ do not enter the domestic market

(ii) $\underline{\varphi} \leq \varphi < \underline{\varphi}_x$ $\rightarrow$ enter the domestic market but do not export

(iii) $\varphi \geq \underline{\varphi}_x$ $\rightarrow$ enter both domestic and foreign market

- <u>Probability of Exporting</u>: Conditional on survival, the probability of exporting is simply

$$prob_x = \frac{1 - G(\underline{\varphi}_x)}{1 - G(\underline{\varphi})}$$

This also means that in equilibrium the mass of firms that exports is

$$M_x = prob_x \cdot M$$

Hence mass of varieties available for consumption is

$$M_t = M + M_x$$

<u>Average profit of a firm</u>: Conditional on survival a firm's average profit is

$$\Pi(\tilde{\varphi}) = \Pi_d(\tilde{\varphi}) + \Pi_x(\tilde{\varphi}_x) \cdot prob_x$$

We know that

$$\Pi_d(\underline{\varphi}) = 0 \Rightarrow r(\underline{\varphi}) = \sigma f$$

And since $\frac{r_d(\tilde{\varphi})}{r(\underline{\varphi})} = \left(\frac{\tilde{\varphi}(\underline{\varphi})}{\underline{\varphi}}\right)^{\sigma-1}$

$$\begin{aligned} \Rightarrow \quad r_d(\tilde{\varphi}) &= \left(\frac{\tilde{\varphi}(\underline{\varphi})}{\underline{\varphi}}\right)^{\sigma-1} \cdot \sigma f \\ \therefore \quad \Pi_d(\tilde{\varphi}) &= \left[\left(\frac{\tilde{\varphi}(\underline{\varphi})}{\underline{\varphi}}\right)^{\sigma-1} - 1\right] f \end{aligned}$$

Similarly, among the firms that export

$$\Pi_x(\tilde{\varphi_x}) = \left[\left(\frac{\tilde{\varphi_x}(\underline{\varphi}_x)}{\underline{\varphi}_x}\right)^{\sigma-1} - 1\right] f_x$$

Thus,

$$\text{(ZCP)} \;:\; \Pi(\tilde{\varphi}) = \left[\left(\frac{\tilde{\varphi}(\underline{\varphi})}{\underline{\varphi}}\right)^{\sigma-1} - 1\right] f + prob_x \cdot \left[\left(\frac{\tilde{\varphi_x}(\underline{\varphi}_x)}{\underline{\varphi}_x}\right)^{\sigma-1} - 1\right] f_x$$

where

$$prob_x = \frac{1 - G(\underline{\varphi}_x)}{1 - G(\underline{\varphi})} \tag{a}$$

And since

$$\begin{aligned} \frac{r_x(\underline{\varphi}_x)}{r_d(\underline{\varphi})} &= \tau^{1-\sigma}\left(\frac{\underline{\varphi}_x}{\underline{\varphi}}\right)^{\sigma-1} = \frac{f_x}{f} \\ \Rightarrow \qquad \underline{\varphi}_x &= \tau\left(\frac{f_x}{f}\right)^{\frac{1}{\sigma-1}} \cdot \underline{\varphi} \end{aligned} \tag{b}$$

- Free entry: due to free entry, the value from entry is driven to zero in equilibrium

$$V^e = \underbrace{(1 - G(\underline{\varphi}))}_{\substack{\text{Probability} \\ \text{of entry}}} \cdot \underbrace{\Pi(\underline{\varphi})}_{\substack{\text{Ex-ante profits} \\ \text{conditional on} \\ \text{which include} \\ \text{expected profits} \\ \text{from exporting}}} - \underbrace{f_e}_{\substack{\text{sunk cost} \\ \text{of entry}}}$$

$$\therefore V^e = 0 \;\Rightarrow \Pi(\tilde{\varphi}) = \frac{f_e}{1 - G(\underline{\varphi})} \tag{FE}$$

- Labor Market Clearing:

$$\begin{aligned} L &= R \\ \Rightarrow \quad L &= M \cdot r(\tilde{\varphi}) \\ \Rightarrow \quad L &= M\left(r_d(\tilde{\varphi}) + r_x(\tilde{\varphi}_x) \cdot prob_x\right) \\ L &= M\left(\left(\frac{\tilde{\varphi}(\underline{\varphi})}{\underline{\varphi}}\right)^{\sigma-1} \sigma f + prob_x(\left(\frac{\tilde{\varphi_x}(\underline{\varphi}_x)}{\underline{\varphi}_x}\right)^{\sigma-1} \sigma f_x\right) \end{aligned}$$

$$\begin{array}{llcl} Since & \Pi(\tilde{\varphi}) & = & \left[\left(\frac{\tilde{\varphi}(\underline{\varphi})}{\underline{\varphi}}\right)^{\sigma-1} - 1\right] f + prob_x \left[\left(\frac{\tilde{\varphi}_x(\underline{\varphi}_x)}{\underline{\varphi}_x}\right)^{\sigma-1} - 1\right] f_x \\ \Rightarrow & \sigma\Pi(\tilde{\varphi}) & = & \left(\frac{\tilde{\varphi}(\underline{\varphi})}{\underline{\varphi}}\right)^{\sigma-1} \sigma f + prob_x (\left(\frac{\tilde{\varphi}_x(\underline{\varphi}_x)}{\underline{\varphi}_x}\right)^{\sigma-1} \sigma f_x + \sigma(f + prob_x f_x) \\ \Rightarrow & \sigma\left[\Pi(\tilde{\varphi}) + f + prob_x f_x\right] & = & \left(\frac{\tilde{\varphi}(\underline{\varphi})}{\underline{\varphi}}\right)^{\sigma-1} \sigma f + prob_x (\left(\frac{\tilde{\varphi}_x(\underline{\varphi}_x)}{\underline{\varphi}_x}\right)^{\sigma-1} \sigma f_x \end{array}$$

$$\therefore M = \frac{L}{\sigma(\Pi(\tilde{\varphi}) + f + prob_x f_x)} \qquad \text{(LMC)}$$

- Equilibrium: is characterized by the following conditions

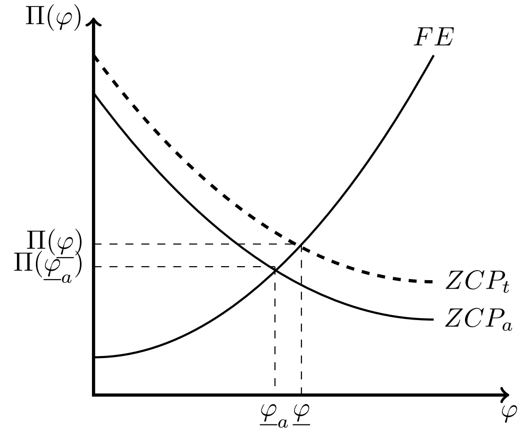
(ZCP) $\Pi(\tilde{\varphi}) = \left[(\left(\frac{\tilde{\varphi}(\underline{\varphi})}{\underline{\varphi}}\right)^{\sigma-1} - 1\right] f + prob_x \left[\left(\frac{\tilde{\varphi}_x(\underline{\varphi}_x)}{\underline{\varphi}_x}\right)^{\sigma-1} - 1\right] f_x$

$$\begin{array}{lcl} \text{where} \quad prob_x & = & \frac{1-G(\underline{\varphi}_x)}{1-G(\underline{\varphi})} \\ \& \qquad \underline{\varphi}_x & = & \tau\left(\frac{f_x}{f}\right)^{\frac{1}{\sigma-1}} \cdot \underline{\varphi} \end{array}$$

(FE) $\Pi(\tilde{\varphi}) = \frac{f_e}{1-G(\underline{\varphi})}$

(LMC) $M = \frac{L}{\sigma(\Pi(\tilde{\varphi})+f+prob_x f_x)}$

- Effect of trade:
- Denote autarky variables with subscript '$a$'
- One can see that the FE condition is identical for autarky and for the trade equilibrium.
- The ZCP condition changes from autarky to trade - the ZCP curve shifts up because the firms that survive on average make higher profits.




Thus, under trade more productive firms survive and, conditional on survival, average profits are higher.

- Thus the least productive firms with $\underline{\varphi}_a < \varphi < \underline{\varphi}$ can no longer earn positive profits in the new trade equilibrium and therefore exit.
- Since

Autarky: $M_a = \frac{L}{\sigma(\Pi(\underline{\varphi}_a + f))}$

Trade: $M = \frac{L}{\sigma(\Pi(\underline{\varphi} + f + prob_x x f_x))}$

$\therefore$ the size of the economy is the same, L; $\Rightarrow M < M_a$

- However, the total varieties under trade is <u>usually</u>

$$M_t = M + M_x > M_a$$

- <u>Reallocation of Market Shares & Profits Across firms</u>
- Consider a firm with $\varphi > \underline{\varphi}_a$ and analyze its performance before and after trade. Let $r_a(\varphi) > 0$ & $\Pi_a(\varphi) \geq 0$ denote firm's revenue and profits in autarky.
- Importantly, both in autarky and in trade, size of an economy $L = R \rightarrow$ aggregate revenue are unchanged.

$$\Rightarrow \quad \underbrace{\frac{r_a(\varphi)}{R}}_{\substack{\text{market share}\\ \text{in autarky}}} \quad \& \quad \underbrace{\frac{r_d(\varphi)}{R}}_{\substack{\text{marker share in}\\ \text{domestic industry}\\ \text{under trade}}}$$

- We can show that

$$r_d(\varphi) < r_a(\varphi) < \underbrace{r_d(\varphi) + r_x(\varphi)}_{(1+\tau^{1-\sigma})r_d(\varphi)}$$

In autarky

$$r_a(\varphi) = \left(\frac{\varphi}{\underline{\varphi}_a}\right)^{\sigma-1} \sigma f \qquad (\forall \varphi \geq \underline{\varphi}_a)$$

In trade

$$r_d(\varphi) = \left(\frac{\varphi}{\underline{\varphi}}\right)^{\sigma-1} \sigma f \qquad (\forall \varphi \geq \underline{\varphi})$$

Since $\underline{\varphi}_a < \underline{\varphi} \Rightarrow r_d(\varphi) < r_a(\varphi)$

↳ This means that all firms incur a loss in domestic sales under trade. Thus, if a firm does not export it incurs a total revenue loss and hence a loss in market share as well.

Now $r_d(\varphi) + r_x(\varphi) = (1 + \tau^{1-\sigma})r_d(\varphi)$

- It can be shown that $(1+\tau^{1-\sigma})r_d(\varphi) \downarrow$ as $\tau \uparrow$.
  Autarky is the limiting case where $\tau \to \infty \Rightarrow \tau^{1-\sigma} \to 0$

$$\begin{array}{ll} \Rightarrow & r_a(\varphi) = \lim_{\tau\to\infty} r_d(\varphi) = \lim_{\tau\to\infty}(1+\tau^{1-\sigma})r_d(\varphi) \\ \Rightarrow & r_a(\varphi) < (1+\tau^{1-\sigma})r_d(\varphi) \end{array}$$

  Thus, a firm that exports more than makes up for the loss of domestic sales with export sales and increases its total revenues.

- Thus exporters $\uparrow$ their share of $R$ while others $\downarrow$ their share of $R \to$ polarization of revenues towards exporters.

$$\begin{aligned} \Delta\Pi(\varphi) &= \Pi(\varphi) - \Pi_a(\varphi) \\ &= \tfrac{r_d(\varphi)+r_x(\varphi)}{\sigma} - f - f_x - \left(\tfrac{r_a(\varphi)}{\sigma} - f\right) \\ &= \tfrac{(1+\tau^{1-\sigma})}{\sigma} r_d(\varphi) - \tfrac{r_a(\varphi)}{\sigma} - f_x \\ &= \tfrac{(1+\tau^{1-\sigma})}{\sigma} \cdot \left(\tfrac{\varphi}{\underline{\varphi}}\right)^{\sigma-1} \cdot \sigma f - f_x \\ &= \varphi^{\sigma-1} \cdot f \left[\tfrac{1+\tau^{1-\sigma}}{\underline{\varphi}^{\sigma-1}} - \tfrac{1}{\underline{\varphi}_a^{\sigma-1}}\right] - f_x \end{aligned}$$

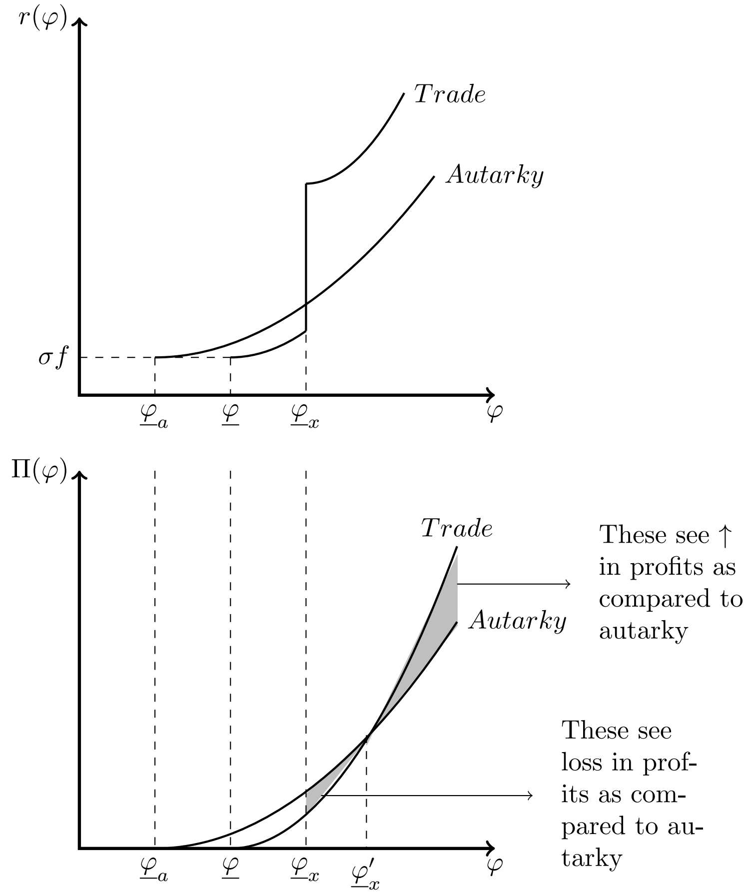
- If a firm does not export, its revenue is lower $\Rightarrow$ the profits must decrease.

- For an exporting firm, the direction of change is not so obvious since the revenues $\uparrow$ but there is n additional fixed cost of exporting $f_x$. ↳ The term in the bracket is positive because $r_d(\varphi) + r_x(\varphi) > r_a(\varphi)$ for all $\varphi > \underline{\varphi}$.

  Since, for the cut-off exporter $r_x(\varphi) = 0$ and $r_d(\varphi) < r_a(\varphi)$
  $\Rightarrow \Delta\Pi(\underline{\varphi}) < 0$ i.e., the change in profit for the cut-off exporter is negative.

  ↳ $\Delta\Pi(\varphi) = \varphi^{\sigma-1} f \left[\frac{1+\tau^{1-\sigma}}{\underline{\varphi}} - \frac{1}{\underline{\varphi}_a}\right] - f_x$
  $\frac{\Delta\Pi(\varphi)}{\partial\varphi} > 0$
  Thus, the change in profit from autarky is increasing in firm productivity. This means that the firms are partitioned in two groups:

  - Those who lose profits
  - Those who gain profits




- Exposure to trade generates Darwinian evolution
  ↳ most efficient firms thrive and grow - they export and increase both market share $\left(\frac{r(\varphi)}{R}\right)$ and profits $(\Pi(\varphi))$.
  ↳ Some less efficient firms still export and increase their market share but incur a profit loss.
  ↳ Some even less efficient firms remain in the industry but do not export and incur losses of both market share and profit.
  ↳ Least efficient firms are forced to exit.

- <u>How does trade affect the distribution of firms</u> Two potential channels:

  (i) <u>Increase in product market competition</u>- firms face an increasing number of competitors.
  ↳ But this channel is not operational here due to the CES demand structure. ↳ With CES the price elasticity of demand does not get affected by the number or prices of competing varieties.

(ii) Increase in demand for inputs (labor): ↳ with trade, exporting provides new opportunities for profits only to the most productive firms who can afford to pay the entry cost of exporting. ↳ These higher potential profits increase entry. ↳ Increased entry increases demand for labor. ↳ Most productive firms whose revenue is increasing also demand more labor. ↳ This increases the real wage and forces the least productive firms to exit.

– Under trade, therefore, average productivity of economy in higher.

**Note:** We have not explained how is the entry cost, $f_e$, of firms who exit after drawing their $\varphi$ paid. The implicit assumption is that the profits of firms who survive and produce are exactly equal to the total entry cost paid for the exiting firms. Think of this as a mutual fund/venture capital fund which in equilibrium breaks even by paying for its loss making stocks/projects from the profits of its profit making stocks/projects.