

Solution to Homework 3

Monopolistic Competition and Increasing Returns

ECO-13101 Economía Internacional I (International Trade Theory)*

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Question 1: Increasing Returns and Pro-Competitive Gains

1. The profit maximization problem of the monopolist is

$$\max_{\{Q\}} (a - bQ) \cdot Q - F - cQ \ .$$

The first-order condition with-respect-to Q is

$$a - 2bQ - c = 0 \ ,$$

$$Q = \frac{a - c}{2b} \ , \tag{1}$$

Therefore, the monopoly price is given by:

$$P = a - b \frac{(a - c)}{2b} = \frac{a + c}{2} \tag{2}$$

Thus,

$$P - c = \frac{a + c}{2} - c = \frac{a - c}{2} \ .$$

This is positive since $a > c$. This shows that the price is greater than the marginal cost, which is because the monopolist has market power and charges a markup which results in a price higher than the marginal cost.

2. The Foreign oil producer will have an incentive to sell oil in the Home market. This is because by doing so, the producer exploit economies of scale, i.e. spread the fixed cost over a larger output, thereby reducing the average cost of production.
3. The profit maximization problem of the Home firm is

$$\max_{\{Q\}} (a - b(Q + Q^*))Q - F - cQ \ .$$

The first-order condition with-respect-to Q is

$$a - b(2Q + Q^*) - c = 0 \ ,$$

$$\Rightarrow Q = \frac{a - c - bQ^*}{2b} \ . \tag{3}$$

4. The profit maximization problem of the Foreign firm is

$$\max_{\{Q^*\}} (a - b(Q + Q^*))Q^* - F - cQ^* .$$

The first-order condition with-respect-to Q^* is

$$a - b(Q + 2Q^*) - c = 0 ,$$

$$\Rightarrow Q^* = \frac{a - c - bQ}{2b} . \quad (4)$$

5. Use Eq. (3) and Eq. (4) to solve for Q and Q^* .

$$Q^* = Q = \frac{a - c}{3b} . \quad (5)$$

Then, the equilibrium price is given by:

$$P = a - b(Q + Q^*) = \frac{a + 2c}{3} \quad (6)$$

6. The consumers benefit from free trade. The total output consumed is higher and the price is lower.

$$Q_{trade} + Q^*_{trade} = \frac{2(a - c)}{3b} = 0.66 \frac{(a - c)}{b} > \frac{a - c}{2b} = 0.5 \frac{(a - c)}{b} = Q_{autarky} ,$$

and

$$P_{trade} = \frac{a + 2c}{3} < \frac{a + c}{2} = P_{autarky} \quad (\text{because } a > c).$$

Question 2: Monopolistic Competition and Gains from Product Diversity

1. The consumer's utility maximization exercise is the following:

$$\begin{aligned} \max_{\{X_i\}_{i=1}^n} & \sum_{i=1}^n X_i^\alpha , \\ \text{s.t.} & \sum_{i=1}^n p_{xi} X_i = w\bar{L} . \end{aligned}$$

The first-order condition with-respect-to a good i is

$$\alpha X_i^{\alpha-1} - \lambda p_{xi} = 0 \quad , \quad i = \{1, \dots, n\} ,$$

where λ is the lagrange multiplier for the consumer. The first-order condition with-respect-to the lagrange multiplier just gives the budget constraint. The first-order condition above implies that

$$X_i = \left[\frac{\lambda p_{xi}}{\alpha} \right]^{1/(\alpha-1)} .$$

Substituting this for X_i in the budget constraint, and noting that labor is the numeraire, i.e. $w = 1$, gives:

$$\begin{aligned} \left[\frac{\lambda}{\alpha} \right]^{1/(\alpha-1)} \sum_{i=1}^n p_{xi}^{\alpha/(\alpha-1)} &= \bar{L} \quad , \\ \Rightarrow \lambda &= \alpha \left[\frac{\bar{L}}{\sum_{i=1}^n p_{xi}^{\alpha/(\alpha-1)}} \right]^{(\alpha-1)} . \end{aligned}$$

Substitute this into the first-order condition obtained with-respect-to X_i .

$$\begin{aligned} \alpha X_i^{\alpha-1} - \alpha \left[\frac{\bar{L}}{\sum_{i=1}^n p_{xi}^{\alpha/(\alpha-1)}} \right]^{(\alpha-1)} p_{xi} &= 0 \quad , \\ \Rightarrow X_i &= \frac{p_{xi}^{1/(\alpha-1)} \bar{L}}{\sum_{i=1}^n p_{xi}^{\alpha/(\alpha-1)}} \quad , \\ \Rightarrow X_i &= \frac{\bar{L}}{p_{xi}^{1/(1-\alpha)} \sum_{i=1}^n p_{xi}^{-\alpha/(1-\alpha)}} . \end{aligned}$$

Let $\sigma = 1/(1 - \alpha)$ and $P = \sum_{j=1}^n p_{xj}^{-\alpha\sigma}$. Then

$$X_i = \frac{\bar{L}}{p_{xi}^\sigma P} \quad , \quad i = \{1, \dots, n\} .$$

This expression gives the demand for good i .

2.

$$\frac{dX_i}{dp_{xi}} = -\sigma \frac{\bar{L}}{P} p_{xi}^{-(\sigma+1)} .$$

Substituting this and the expression obtained for X_i (in the previous part) in the formula for elasticity of demand gives:

$$e_i = - \left[-\sigma \frac{\bar{L}}{P} p_{xi}^{-(\sigma+1)} \cdot \frac{p_{xi}}{\bar{L}/(p_{xi}^\sigma P)} \right] ,$$

$$e_i = \sigma \left[\frac{\bar{L} p_{xi}^{-(\sigma+1)}}{P} \cdot \frac{p_{xi}^{(\sigma+1)} P}{\bar{L}} \right] ,$$

$$\Rightarrow e_i = \sigma = \frac{1}{1 - \alpha} , \quad i = \{1, \dots, n\} .$$

3. The profit function of the firm producing good i is given by $\pi_{xi} = TR_{xi} - TC_{xi}$, where TR_{xi} is the total revenue of the firm and TC_{xi} is the total costs of the firm. The total cost function has been given to us. What we need is to derive the total revenue function. Well, total revenue is nothing but the value of the sales, i.e. quantity sold (what is demanded is sold) times the price at which it sold - $TR_{xi} = p_{xi} X_i$. Therefore, after substituting for X_i , the profit maximization problem is given by:

$$\max_{p_{xi}} \pi_{xi} = p_{xi}^{1-\sigma} \frac{\bar{L}}{P} - F - MC_{xi} \cdot \frac{\bar{L}}{p_{xi}^\sigma P} .$$

Note that choosing a price is equivalent to choosing output since the two are uniquely related through the demand function (the expression for X_i derived in (1)). The first-order condition with-respect-to price is

$$(1 - \sigma) p_{xi}^{-\sigma} \frac{\bar{L}}{P} - MC_{xi} \cdot \left[-\sigma p_{xi}^{-(\sigma+1)} \cdot \frac{\bar{L}}{P} \right] = 0 ,$$

$$\Rightarrow -(\sigma - 1) p_{xi} + \sigma MC_{xi} = 0 ,$$

$$\Rightarrow p_{xi} = \frac{\sigma}{\sigma - 1} MC_{xi} = \frac{1}{\alpha} MC_{xi} , \quad i = \{1, \dots, n\} .$$

Since $0 < \alpha < 1$, it implies that the price is greater than marginal cost. So $1/\alpha$ represents the markup. Notice that

$$e_i = \sigma = \frac{1}{1 - \alpha} \Rightarrow \alpha = 1 - \frac{1}{e_i} \Rightarrow \frac{1}{\alpha} = \frac{e_i}{e_i - 1} .$$

Therefore, the price of good i could also be written as:

$$p_{xi} = \frac{e_i}{e_i - 1} MC_{xi} , \quad i = \{1, \dots, n\} .$$

4. Due to free entry of firms (to produce each good i), the profit of each firm is driven to zero, i.e. $p_{xi} = AC_{xi}$, where AC_{xi} is the average cost of producing good i . The average costs is nothing but TC_{xi}/X_i . Using the expression for price obtained in the previous part, we get

$$\begin{aligned}\frac{1}{\alpha}MC_{xi} &= \frac{TC_{xi}}{X_i} = \frac{F}{X_i} + MC_{xi} \ , \\ \Rightarrow \frac{F}{X_i} &= \frac{1-\alpha}{\alpha}MC_{xi} \ , \\ \Rightarrow X_i &= \frac{\alpha F}{MC_{xi}(1-\alpha)} \ , \ i = \{1, \dots, n\} \ .\end{aligned}$$

5. Substituting the expression for X_i obtained in the previous part in the expression for total cost gives

$$\begin{aligned}TC_{xi} &= F + MC_{xi} \cdot \frac{\alpha F}{MC_{xi}(1-\alpha)} \ , \\ \Rightarrow TC_{xi} &= \frac{1}{1-\alpha}F \ , \ i = \{1, \dots, n\} \ .\end{aligned}$$

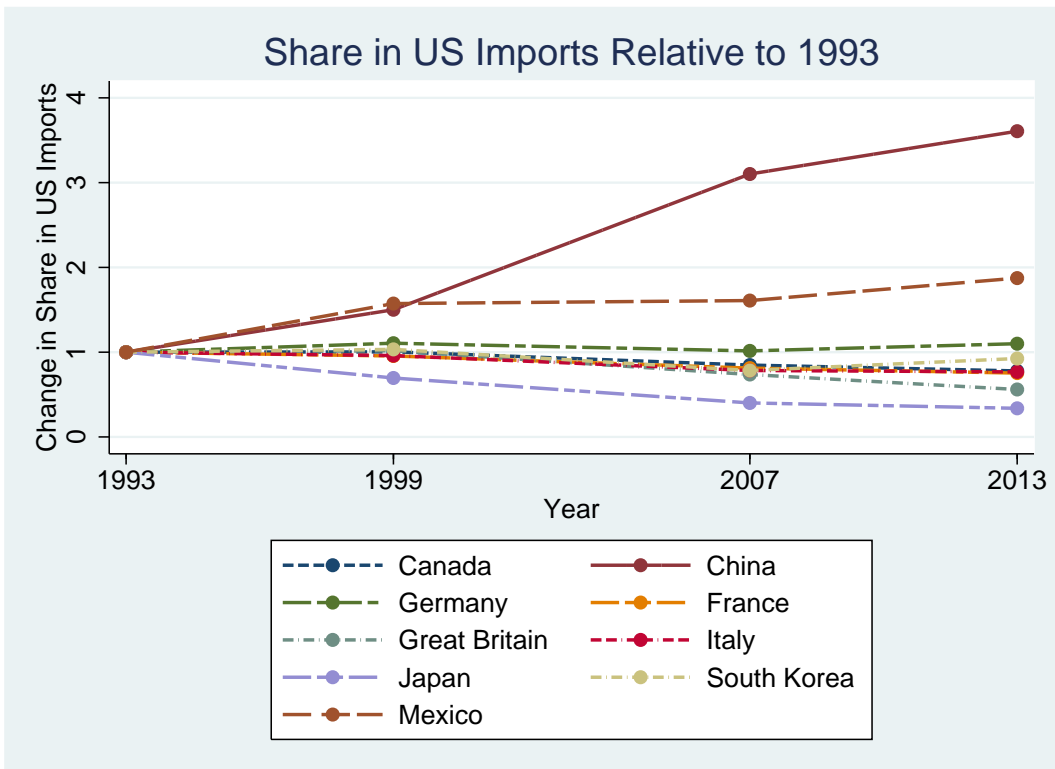
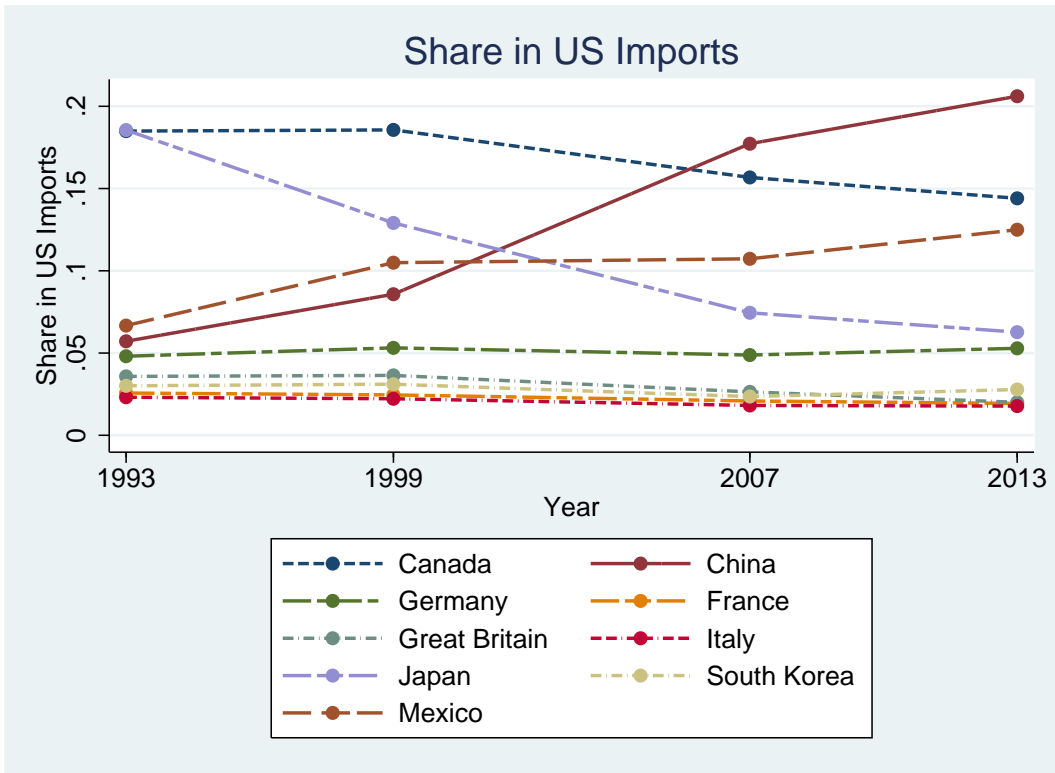
This expression shows that the total cost is the same for every good $i = \{1, \dots, n\}$. Since labor is the only factor, it must be that the total cost of producing the n goods is equal to the total labor cost, i.e. $\sum_{i=1}^n TC_{xi} = w\bar{L}$.

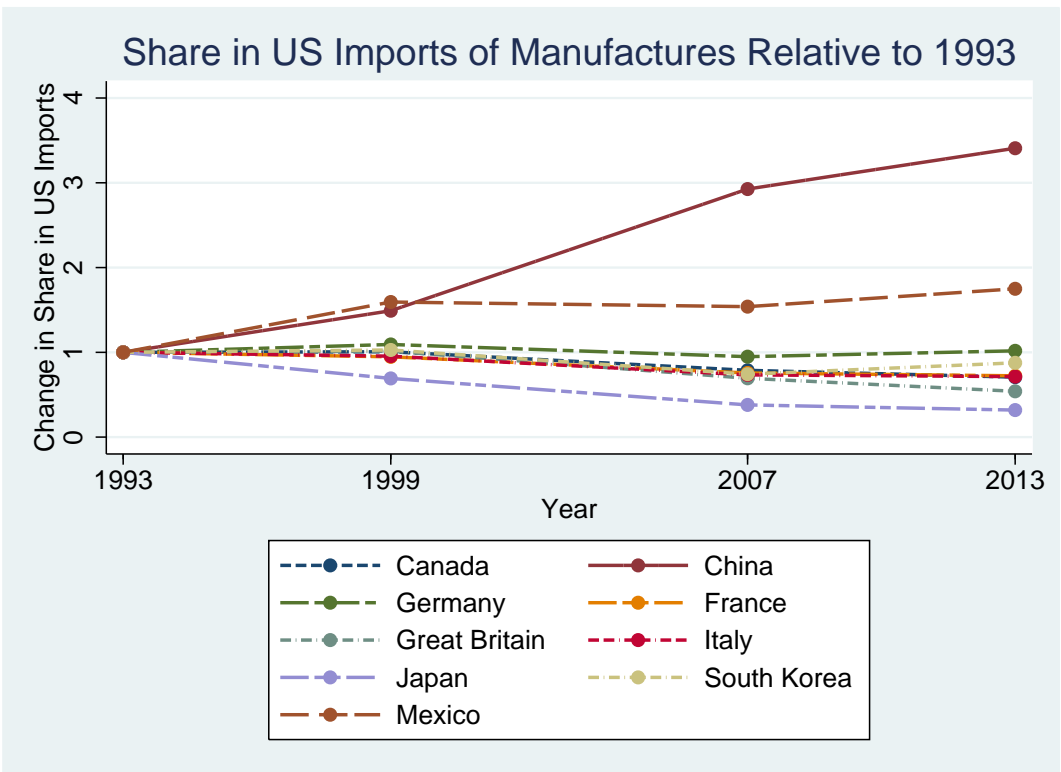
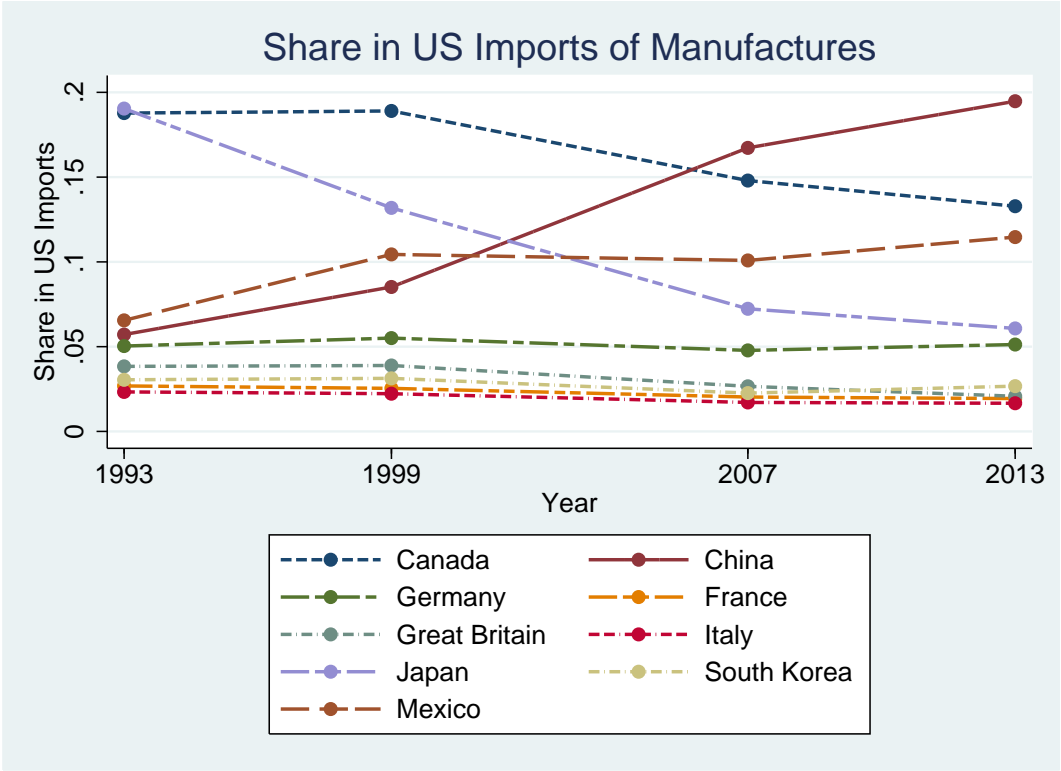
$$\begin{aligned}\sum_{i=1}^n \frac{1}{1-\alpha}F &= w\bar{L} \ , \\ \Rightarrow n \cdot \frac{1}{1-\alpha}F &= \bar{L} \ , \\ \Rightarrow n &= (1-\alpha) \frac{\bar{L}}{F} \ .\end{aligned}$$

Thus, the numbers of good produced in the economy is going to depend on the total labor income relative to the fixed cost (per good).

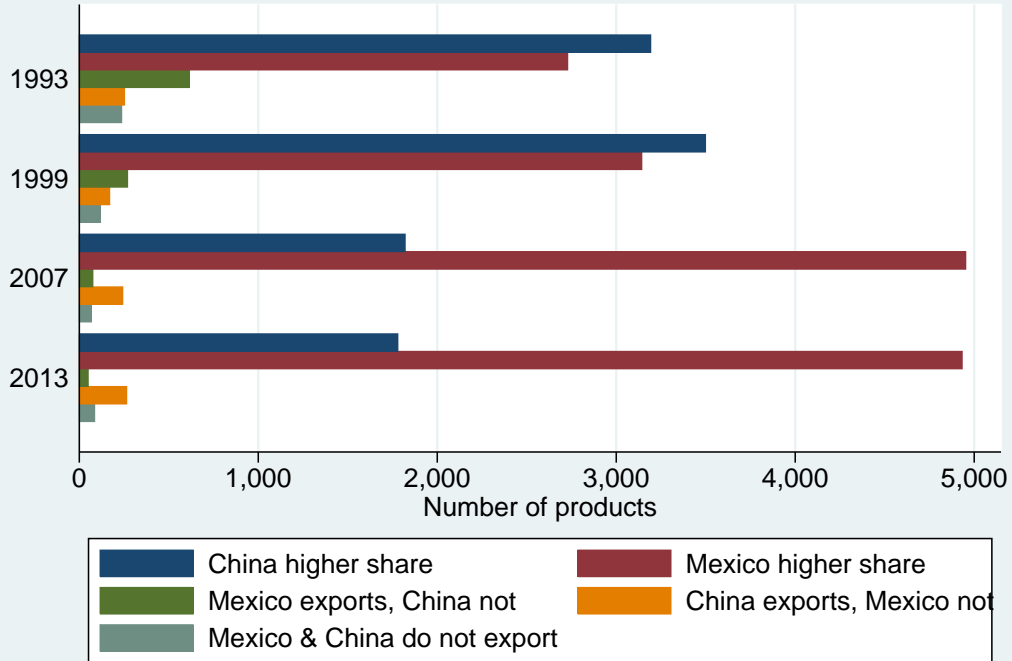
6. Thus, if the endowment of labor doubled, it would increase the number of goods produced in the economy, i.e. increase product diversity, which in turn will cause the utility to increase. This tells us that when two such (identical) economies open up to trade, then the combined economy will have a larger labor endowment (size), $2\bar{L}$, and therefore this will increase the product diversity. Increase in product diversity will increase welfare. Thus, trade allows the two countries to exploit scale economies and therefore increase product diversity.

Question 3: Mexico and its Competitors in the US Market





Mexico versus China – Number of Products Exported



Year	Importer	Exporter	Product Code	Product Description	Share in US Imports	Rank Mexico	Year	Importer	Exporter	Product Code	Product Description	Share in US Imports	Rank China
1993	USA	MEX	3330	Petrol./bitum. oil,crude	8.23%	1	1993	USA	CHN	8942	Childrens toys	5.90%	1
1993	USA	MEX	7812	Pass motor veh exc buses	5.91%	2	1993	USA	CHN	8514	Footwear leather upr nes	4.02%	2
1993	USA	MEX	7731	Insulated wire/opt fibre	4.31%	3	1993	USA	CHN	8513	Rub/plast footwear nes	3.31%	3
1993	USA	MEX	7843	Motor veh part/acces nes	4.24%	4	1993	USA	CHN	8453	Jerseys/pullovers/etc	2.23%	4
1993	USA	MEX	7611	Colour tv receivers	2.98%	5	1993	USA	CHN	8427	Womens/girls blouse wven	1.81%	5
1993	USA	MEX	7725	Elec swithing etc <1000v	2.00%	6	1993	USA	CHN	8944	Festive entertainm artic	1.51%	6
1993	USA	MEX	7649	Telecomms parts/access.	1.61%	7	1993	USA	CHN	8319	Special purpose cases	1.20%	7
1993	USA	MEX	7132	Motor veh. i/c pistn eng	1.46%	8	1993	USA	CHN	8481	Leather clothing/access	1.16%	8
1993	USA	MEX	7621	Vehicle etc radio recvrs	1.22%	9	1993	USA	CHN	8311	Handbags	1.09%	9
1993	USA	MEX	8211	Chairs and seats	1.15%	10	1993	USA	CHN	7622	Battery etc radios	1.05%	10
1999	USA	MEX	7812	Pass motor veh exc buses	10.39%	1	1999	USA	CHN	8942	Childrens toys	8.18%	1
1999	USA	MEX	3330	Petrol./bitum. oil,crude	7.08%	2	1999	USA	CHN	8514	Footwear leather upr nes	4.51%	2
1999	USA	MEX	7731	Insulated wire/opt fibre	5.28%	3	1999	USA	CHN	7599	Office equip parts nes	3.26%	3
1999	USA	MEX	7821	Goods transport vehicles	4.36%	4	1999	USA	CHN	7526	Adp peripheral units	3.01%	4
1999	USA	MEX	7611	Colour tv receivers	4.35%	5	1999	USA	CHN	8513	Rub/plast footwear nes	2.79%	5
1999	USA	MEX	7843	Motor veh part/acces nes	3.79%	6	1999	USA	CHN	8131	Lamps/lighting fittings	2.16%	6
1999	USA	MEX	7643	Radio/tv transmit equip.	2.42%	7	1999	USA	CHN	8944	Festive entertainm artic	2.07%	7
1999	USA	MEX	7526	Adp peripheral units	2.27%	8	1999	USA	CHN	7638	Sound/tv recorders etc	1.81%	8
1999	USA	MEX	8211	Chairs and seats	2.16%	9	1999	USA	CHN	7641	Telephone equipment	1.73%	9
1999	USA	MEX	7599	Office equip parts nes	2.06%	10	1999	USA	CHN	8453	Jerseys/pullovers/etc	1.50%	10
2007	USA	MEX	3330	Petrol./bitum. oil,crude	12.23%	1	2007	USA	CHN	7522	Digital computers	6.94%	1
2007	USA	MEX	7611	Colour tv receivers	7.35%	2	2007	USA	CHN	7643	Radio/tv transmit equip.	5.16%	2
2007	USA	MEX	7812	Pass motor veh exc buses	5.36%	3	2007	USA	CHN	8943	Games equipment	4.06%	3
2007	USA	MEX	7843	Motor veh part/acces nes	4.01%	4	2007	USA	CHN	7599	Office equip parts nes	4.03%	4
2007	USA	MEX	7821	Goods transport vehicles	3.71%	5	2007	USA	CHN	8942	Childrens toys	3.95%	5
2007	USA	MEX	7731	Insulated wire/opt fibre	2.76%	6	2007	USA	CHN	7611	Colour tv receivers	3.68%	6
2007	USA	MEX	7523	Digital processing units	1.68%	7	2007	USA	CHN	7638	Sound/tv recorders etc	3.49%	7
2007	USA	MEX	8211	Chairs and seats	1.51%	8	2007	USA	CHN	8514	Footwear leather upr nes	3.05%	8
2007	USA	MEX	7643	Radio/tv transmit equip.	1.47%	9	2007	USA	CHN	7526	Adp peripheral units	2.96%	9
2007	USA	MEX	7649	Telecomms parts/access.	1.14%	10	2007	USA	CHN	8211	Chairs and seats	2.75%	10
2013	USA	MEX	3330	Petrol./bitum. oil,crude	11.56%	1	2013	USA	CHN	7522	Digital computers	14.74%	1
2013	USA	MEX	7812	Pass motor veh exc buses	7.19%	2	2013	USA	CHN	7643	Radio/tv transmit equip.	14.17%	2
2013	USA	MEX	7843	Motor veh part/acces nes	6.17%	3	2013	USA	CHN	8942	Childrens toys	3.60%	3
2013	USA	MEX	7821	Goods transport vehicles	5.54%	4	2013	USA	CHN	7599	Office equip parts nes	3.30%	4
2013	USA	MEX	7611	Colour tv receivers	4.81%	5	2013	USA	CHN	8211	Chairs and seats	3.20%	5
2013	USA	MEX	7523	Digital processing units	4.26%	6	2013	USA	CHN	7843	Motor veh part/acces nes	2.66%	6
2013	USA	MEX	7731	Insulated wire/opt fibre	3.02%	7	2013	USA	CHN	7611	Colour tv receivers	2.64%	7
2013	USA	MEX	8211	Chairs and seats	2.09%	8	2013	USA	CHN	8514	Footwear leather upr nes	2.59%	8
2013	USA	MEX	7832	Semi-trailer tractors	1.80%	9	2013	USA	CHN	8453	Jerseys/pullovers/etc	2.20%	9
2013	USA	MEX	8722	Medical.surg/vet instrum	1.48%	10	2013	USA	CHN	8131	Lamps/lighting fittings	2.06%	10

Year	Importer	Product Code	Product Description	Mexico share in Us Imports	Mexico Rank	China share in Us Imports	China Rank
1993	USA	3330	Petrol./bitum. oil,crude	8.23%	1	0.45%	32
1993	USA	7812	Pass motor veh exc buses	5.91%	2	0.00%	621
1993	USA	7731	Insulated wire/opt fibre	4.31%	3	0.33%	44
1993	USA	7843	Motor veh part/acces nes	4.24%	4	0.13%	89
1993	USA	7611	Colour tv receivers	2.98%	5	0.19%	70
1993	USA	7725	Elec swithing etc <1000v	2.00%	6	0.23%	61
1993	USA	7649	Telecomms parts/access.	1.61%	7	0.21%	66
1993	USA	7132	Motor veh. i/c pistn eng	1.46%	8	0.00%	607
1993	USA	7621	Vehicle etc radio recvrs	1.22%	9	0.14%	85
1993	USA	8211	Chairs and seats	1.15%	10	0.29%	49
1999	USA	7812	Pass motor veh exc buses	10.39%	1	0.00%	533
1999	USA	3330	Petrol./bitum. oil,crude	7.08%	2	0.08%	150
1999	USA	7731	Insulated wire/opt fibre	5.28%	3	0.75%	26
1999	USA	7821	Goods transport vehicles	4.36%	4	0.00%	716
1999	USA	7611	Colour tv receivers	4.35%	5	0.06%	170
1999	USA	7843	Motor veh part/acces nes	3.79%	6	0.34%	55
1999	USA	7643	Radio/tv transmit equip.	2.42%	7	0.41%	47
1999	USA	7526	Adp peripheral units	2.27%	8	3.01%	4
1999	USA	8211	Chairs and seats	2.16%	9	1.41%	11
1999	USA	7599	Office equip parts nes	2.06%	10	3.26%	3
2007	USA	3330	Petrol./bitum. oil,crude	12.23%	1	0.06%	260
2007	USA	7611	Colour tv receivers	7.35%	2	3.68%	6
2007	USA	7812	Pass motor veh exc buses	5.36%	3	0.09%	199
2007	USA	7843	Motor veh part/acces nes	4.01%	4	1.52%	18
2007	USA	7821	Goods transport vehicles	3.71%	5	0.00%	677
2007	USA	7731	Insulated wire/opt fibre	2.76%	6	1.01%	26
2007	USA	7523	Digital processing units	1.68%	7	2.00%	13
2007	USA	8211	Chairs and seats	1.51%	8	2.75%	10
2007	USA	7643	Radio/tv transmit equip.	1.47%	9	5.16%	2
2007	USA	7649	Telecomms parts/access.	1.14%	10	1.17%	24
2013	USA	3330	Petrol./bitum. oil,crude	11.56%	1	0.02%	432
2013	USA	7812	Pass motor veh exc buses	7.19%	2	0.03%	386
2013	USA	7843	Motor veh part/acces nes	6.17%	3	2.66%	6
2013	USA	7821	Goods transport vehicles	5.54%	4	0.00%	739
2013	USA	7611	Colour tv receivers	4.81%	5	2.64%	7
2013	USA	7523	Digital processing units	4.26%	6	2.01%	11
2013	USA	7731	Insulated wire/opt fibre	3.02%	7	1.36%	24
2013	USA	8211	Chairs and seats	2.09%	8	3.20%	5
2013	USA	8722	Medical.surg/vet instrum	1.48%	10	0.26%	117