ENDOGENOUS CAPITAL UTILIZATION IN A NEOCLASSICAL GROWTH MODEL

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This paper introduces a variable rate of capital utilization and depreciation into a modified Ramsey-type neoclassical growth model via the well-known concept of pure user cost. The optimal utilization rate is found to be determined by the opportunity cost of holding capital or the net real interest rate. As a consequence, this rate may vary in the short run so total services of capital become a control rather than a state variable; furthermore, the introduction of a variable utilization rate yields a slower rate of convergence towards the steady state inducing more persistence in the transitional dynamics. In order to illustrate how the endogenous choice of utilization acts on the system, some simulations are carried out, including the transition period when there is a temporary fall in the exogenous real interest rate. (JEL E13)

INTRODUCTION

In the standard theory of capital and factor demand the rate of physical depreciation is usually taken to be constant; nonetheless, it has long been recognized that depreciation may be subject to choice by the users of capital. The idea that capital depreciates faster when used more intensively has been around at least since Marshall [1922] and was tackled by Keynes [1935] through the concept of "pure user cost." Capital may be used with varying degrees of intensity and the cost of such use is borne through increased depreciation of the good, commonly known as "wear and tear". Examples of this line of research are Calvo [1975], Auernheimer [1986], Diewert [1986], Bischoff and Kokkelenberg [1987], Johnson [1994] and references therein.

There is a second strand in the literature concerning the choice of an optimal depreciation rate, developed independently despite addressing the same problem. This is related to the case in which depreciation is "embodied" in the capital good at the time at which it is produced. The depreciation rate is then interpreted as a measure of durability or "quality". In this case, both the cost of production and the market price of the good are decreasing functions of the depreciation rate and it is up to the producer to choose the optimal depreciation rate. The question has been studied in the literature on "consumer durable" goods, in particular after an important paper by Swan [1970] showing that optimal durability is independent of demand considerations and ultimately determined by the real interest rate. Auernheimer and Saving [1977] extend Swan's result to a more general setup, in which the firm is subject to adjustment costs.

Greenwood, Hercowitz and Huffman [1988] are the first to formally model the impact of a variable utilization rate of capital to explain variations in output. Theirs is a fairly standard real business cycle model (RBC) where the cycles are generated by way of shocks to investment. More recently DeJong, Ingram and Whiteman [1995], Burnside, Eichenbaum and Rebelo [1995] and Licandro and Puch [1996], among others, have studied the effect of a variable utilization rate in various RBC models.

This paper provides a qualitative analysis of the variable utilization rate of capital assumption. In order to do this, it assumes a continuous time environment using one of the many variations of the typical one sector, Ramsey growth model. To concentrate in the production or "supply side" of the economy, the utilization rate and investment decisions, agents can hold an alternative asset yielding a fixed rate of return. The typical "small open economy" with a foreign asset and a given world interest rate is, perhaps, the best example of such a case. To have a dynamic setup where adjustment is not instantaneous, the usual procedure of introducing adjustment costs on investment is adopted.

The assumption of a variable utilization rate of capital, not only turns the capital input into a control variable (total services of capital), but it also affects the slope of the saddle path and the convergence rate. These points are illustrated with some simulations where the model is compared to one with a fixed utilization rate of capital. Finally, a temporary fall (rise) in the world interest rate is considered in order to highlight the differences between the two models during the adjustment process.

THE MODEL

Notation and Basics

The usual one sector neoclassical growth model is adopted, except that (i) there are costs associated with the level of (gross) investment, and (ii) agents can hold either physical capital or an alternative asset yielding a constant rate of return. This particular version of the standard growth model is well known, see for example Barro and Sala-i-Martin [1994, p. 96]. The variable utilization rate of capital is introduced, as specified below, within this specialized version.

The model is that of a small open economy populated by infinitely lived representative agents. For simplicity, the population is constant and equal to the labor force which is normalized to unity. There is one good that may take the form of capital, k, or consumption, c. Production depends on two inputs: the fixed level of labor and services of the capital stock, and is subject to constant returns to scale. Let s denote services per unit of capital referred to as the “utilization rate of capital”, so that $S = sk$ stands for the total flow of services rendered by the capital stock. Total services, S, may be transformed into output by means of a production technology, f, yielding $Q = f(S)$, where f is a twice differentiable increasing and concave function. Within this context, capital may be thought of as machines that may be operated or used at a variable rate s, this utilization rate may be interpreted as the “speed of operation” or “intensity of use”.

Assume that capital depreciates at a rate, $\delta = \delta(s)$, that is a twice differentiable increasing and convex function of s satisfying $\delta(0) = \delta_0 > 0$. This function captures the “user cost” concept of capital utilization, in the sense that capital wears out faster when used more intensively. The condition, $\delta(0) = \delta_0$, implies that, even when not in use, capital still depreciates at the rate δ_0. The representative agent in this economy is a rational individual endowed with perfect foresight, who derives utility from consumption by way of a utility function, $u(c)$. The function u is, as usual, twice differentiable, increasing and concave. The agent may hold physical capital and also a foreign asset, a, with a constant rate of return, r, equal to the world real interest rate. This asset may be negative;

hence, debt is allowed. Adjustment costs on investment are introduced so that it is impossible for individuals to adjust their portfolios discretely at any given time; thus, both capital and foreign assets are state variables determined by past history. *I* stands for the rate of gross investment and *h(I)* denotes an increasing *per unit investment cost function*, such that total cost (*Ih*) is convex [1]

Solution

The typical individual maximizes his lifetime utility of consumption subject to the usual flow budget constraint and given the capital evolution equation, i.e.,

$$\max \int_0^\infty u(c)e^{-\rho t}dt$$

subject to,

$$da/dt = f(sk) + ar - c - I[1 + h(I)], \tag{1}$$

$$dk/dt = I - \delta(s)k, \tag{2}$$

given some initial stock holdings k_0 and a_0.

In the above expression, ρ stands for the individual's time preference rate and for future reference, a superscript, %, on a variable will denote its proportional rate of change, that is $x^{\%} = (dx/dt)/x$. It is assumed that $r = \rho$.[2]

The current value Hamiltonian for the above problem can be expressed by

$$H = u(c) + \lambda_1(da/dt) + \lambda_2(dk/dt),$$

where λ_1 and λ_2 are the costate variables. Letting $p \equiv \lambda_2/\lambda_1$ so that p is the real shadow price of capital, the first order necessary conditions are given by

$$\lambda_1 = u', \tag{3}$$

$$d\lambda_1 / dt = 0, \tag{4}$$

$$f'(sk) = p\delta'(s), \tag{5}$$

$$p = 1 + h(I) + Ih'(I), \tag{6}$$

$$dp/dt = -sf'(sk) + p[r + \delta(s)] \tag{7}$$

and the usual transversality requirements.

Equations (3) and (4) imply a constant level of consumption. Expression (5) implicitly determines a function $s = s(k,p)$. Equation (6) implicitly determines investment as an increasing function of p, $I = I(p)$, which may be interpreted as an "investment supply" schedule. Notice that expression (6) says that at all times the price of capital is equal to the marginal cost of transforming the consumption good into the capital good, through the investment process.

Substituting the above expressions for s and I into (2) and (7) and linearizing the resulting system of differential equations around the long run equilibrium point (k^*,p^*) one obtains

$$\begin{pmatrix} dk/dt \\ dp/dt \end{pmatrix} = \begin{pmatrix} -(\delta + k\delta' s_k) & I' - k\delta' s_p \\ -ps\delta'' s_k & -ps\delta'' s_p \end{pmatrix} \begin{pmatrix} k - k^* \\ p - p^* \end{pmatrix}, \tag{8}$$

where all the functions are evaluated at their long run steady state values. The determinant of the system matrix is negative, so saddle path stability always exists around the equilibrium point. The saddle path is a negatively sloped path towards equilibrium as depicted in figure 1.

Assume that there is a system with a fixed rate of utilization and depreciation and another system with flexibility in the choice of *s*, both sharing a common steady state. In order to detect the differences between the transitional dynamics of the two systems, it is necessary to know how flexibility affects the saddle path. It is not hard to see that when flexibility is introduced, a “flatter” saddle path, and a lower speed of adjustment towards the steady state arise. A proof of this statement is included in the appendix.

The constant level of consumption, c^*, is determined from the lifetime budget constraint and the transversality condition as

$$c^* = a_0 r + r\int_0^\infty [f(sk) - I(1 + h(I)]e^{-rt}dt,$$

where *s*, *k* and *I* are evaluated along their optimal path. The flow *da/dt* + *ar* (i.e., foreign asset accumulation) is then determined at all times by the difference between net output, *f(sk)* - *I(1+h(I))*, and consumption. Finally, notice that equations (5) and (7) yield the relationship

$$s\delta'(s) - \delta(s) = r - p^{\%}. \tag{9}$$

The interpretation of (9) is straightforward: if *s* and *k* are thought of as the two factors needed to produce a total amount of services *S*, then the representative agent’s problem is to minimize the total cost of producing a given level, *S*, of total services. This problem can be stated as

$$\min\{(r - p^{\%})k + \delta(s)k\} \text{ subject to } sk = S.$$

The first term in this cost function represents the opportunity cost of capital and the second term the "depreciation" cost. The optimal *s* solving the above minimization problem must satisfy a first order condition which is identical to (9). Condition (9) is equivalent to that obtained in Auernheimer [1986] for the optimal depreciation rate in the industry. It is also equivalent to the condition derived in Swan [1970] for the case of an optimal embedded depreciation rate for durable consumer goods. Johnson [1994], gives the following interpretation of the expression $s\delta' - \delta$: Given a fixed quantity of total services, *S*, consider the derivative $d\delta k/dk = \delta - s\delta'$. The first part of this expression represents the increase in total depreciation, δk, due to the marginal unit of capital, the second is the reduction in depreciation of the total capital stock due to a lower utilization rate. Thus, $s\delta' - \delta$ represents the return of the marginal unit of capital measured in depreciation savings.

Equation (9) implies that the optimal level of services per unit of capital, *s*, is determined only by the net real interest rate, $r - p^{\%}$, and the functional form $\delta(s)$. Given that *r* is an exogenous constant, it is the *rate of change*, $p^{\%}$, that is accountable for any change in *s*. On the other hand expression (6) implies that investment is determined at all times by the *level* of *p*.

CAPITAL UTILIZATION ALONG THE SADDLE PATH

Consider the adjustment of the system for a given initial level of the capital stock. If the initial capital stock, k_0, is less than that of the long run steady state, then the economy is growing. Initial investment is higher than its equilibrium value and falls towards it while capital converges to its long run steady state. During the adjustment process, the net real interest rate is greater than *r* and falling; thus, capital is used more intensively in this period.[3] In other words, in order to produce a given amount of total services, a substitution away from capital and into utilization takes place. A higher initial capital stock with a contracting economy yields completely symmetric results.

It is now straightforward to analyze the effects of sudden changes in the capital stock, such as a destruction of a portion Δk, given that the system is initially resting at its long run steady state (k^*, p^*). At the time of the shock ("shock", in this context, refers to unexpected changes in parameters or initial conditions), consumption falls and remains at that level from this time onwards. This is caused by a permanent reduction in the present value of net domestic output caused by the destruction of capital. This permanent fall in consumption is a direct consequence of the open economy assumption. In the absence of the term *ar* in the budget constraint, consumption is, at all times, equal to net output and must eventually return to its original level.

In the above considerations, a variable utilization rate causes the initial rise in *p*, and hence in *I*, to be smaller in magnitude when compared to the fixed *s* and δ case due to the flatter saddle path. If the cost of investing is expected to fall in the future, a variable utilization rate of capital allows agents to postpone investment. Individuals may initially invest less than they would be able to without a variable utilization rate, offsetting their present lower capital stock with a higher utilization rate. Furthermore, the convergence towards the steady state takes place at a slower rate when a variable utilization rate is allowed, in other words, there is more persistence in the shock. This observation is important since convergence in this type of model is generally associated with long term trend rather than transition in the short run.

SOME SIMULATIONS

To further examine what a variable utilization rate of capital contributes to the model, a numerical simulation is done for various values of the parameters. In each case two systems are considered: one with a variable and the other one with a fixed utilization rate; furthermore, both systems converge to the same steady state. The following specific functional forms and values of the parameters are used throughout:

$$f(sk) = (sk)^{\alpha}, \tag{10}$$

$$\delta(s) = \delta_0 + s^{\beta}, \tag{11}$$

$$h(I) = bI \tag{12}$$

r = 0.06, δ_0 = 0.01; the other parameters will vary as specified in tables 1 through 3. As usual, the speed of convergence towards the steady state is given by the magnitude of the negative eigenvalue of the matrix (8).

Each of the tables corresponds to a given value of the exponent β in (11). The greater the values of β, the more the system resembles the usual fixed utilization rate case. Two values of the exponent α are considered: 0.3 and 0.75; these represent the share of total services of capital in gross output. The two possible values are justified in Barro and Sala-i-Martin [1995, p. 38] depending on the concept of capital being used. The 0.3 value corresponds to a narrow concept of capital as plant and equipment and the higher 0.75 corresponds to a wider concept that includes human as well as physical capital. The coefficient b in equation (12) is varied from 1 to 5 in order to account for different levels of adjustment costs. In each table, the values indicated by δ^*, p^* and k^* represent the steady state values and the last two columns give, respectively, the convergence rates, λ_v and λ_f for variable and fixed utilization rates.

Blanchard, Changyong and Lawrence [1993] estimate the value of p to be in the neighborhood of 1.5. Barro and Sala -i- Martin [1995, pp.124-125] mention that higher values of p may be plausible when adopting the wider definition of capital; the reason being that human capital may have a very high adjustment cost. In the simulations, the parameter b takes values from 1 to 5 in order to obtain different rates of convergence. The value of p^* is dependent on b, and in most cases it stays within *reasonable* ranges. The difference between rates of convergence goes from a striking 10%-18% in table 1 to a negligible 0.1%-0.5% in table 3. This is to be expected since as the parameter, β, takes higher values the models differ less from each other.

Lower convergence rates assign more importance to the transitional dynamics; furthermore, in a discretized version of the model where uncertainty is explicitly introduced, a lower rate of convergence would be related to higher *persistence* of the various stochastic shocks. Licandro and Puch (1996) recognize the importance of a variable utilization rate of capital as a persistence factor of aggregate shocks.

THE RESPONSE TO A TRANSITORY CHANGE IN THE INTEREST RATE

In order to illustrate the difference in response to parameter changes that a variable utilization rate may bring about, changes in the exogenously given real interest rate are considered. Starting from an initial long run equilibrium, assume that at time $t = 0$ there is an unanticipated fall in the constant real interest rate expected to end at a future known time $t = \tau$. In the simulation, the initial (and final) real interest rate is assumed to be 0.06, with a transitory fall to 0.04 lasting ten periods.

The model assumes at the outset that $r = \rho$, i.e., that the interest rate is equal to the rate of time preference. In this section, the inequality, $r < \rho$, holds temporarily and during that interval consumption falls. Although the analysis of the behavior on the production side (the capital stock, its price, investment and intensity of use) is not affected, the initial level of foreign assets is assumed to be sufficiently large so that no solvency condition is violated.

Figures 2 through 5 show the comparative behavior of the various magnitudes for the case of variable intensity of use and for the case of a fixed intensity of use equal to the initial equilibrium value. Figures 2 and 3 show the behavior of the price and the stock of capital, respectively. The first obvious observation is that flexibility brings about both a wider fluctuation and longer persistence in both of these quantities. The effects on the price of capital are traced by the changes in gross investment. These are reflected also in the behavior of net investment, except that they are importantly magnified due to what happens to the rate of depreciation during the transition. This becomes clear in figure 4, depicting the path of the utilization rate. Graphs for the rate of depreciation and the net real interest rate, not shown here, would also trace the path of the rate of utilization.

The two most important conclusions, though, are reflected in the graph of figure 5. Flexibility in the choice of utilization causes output to drop due to the initial fall in intensity of use; thus, a fall in the interest rate turns out to be initially contractionary. The conventional wisdom of an expansionary effect of a fall in the rate of interest is reflected for the case in which intensity of use and depreciation are fixed: during the interim period for which the interest rate is lower, output is always higher. When flexibility in the choice of utilization is assumed, this is hardly the case; rather, the effect is initially in the opposite direction, and of comparatively much larger magnitude. Of course, the expansionary effect on investment is not only preserved, but also augmented. It should be noted that this result is not the consequence of the fall in the interest rate being transitory. Simulations for the case of a permanent change show the same qualitative results for the initial effects.

The analysis of a permanent fall in the interest rate, starting from an initial benchmark case of the rates of interest and time preference being equal, presents the problem of consumption permanently falling. As mentioned before one could introduce bounds on international borrowing to avoid this. The model could have started from a situation for which $r > \rho$, with a level of consumption and assets rising forever but then the assumption of a “small country” runs into trouble.

CONCLUDING REMARKS

The previous sections, showed the importance of taking into consideration the pure user cost of capital at the general equilibrium level. Two previous models (Auernheimer [1986] and Calvo [1975]) were extended in order to obtain an optimality condition for the utilization rate of the capital stock, or equivalently, for an optimal depreciation rate. The optimal level of services per unit of capital, turned out to be determined, at each point in time, exclusively by the net real interest rate. A consequence of the model is that flexibility in the utilization of capital reduces the speed of convergence towards the steady state and the slope of the saddle path. This induces more persistence in reaction to changes in the parameters and an ambiguous effect on the magnitude or amplitude of the changes.

When analyzing the effect of a temporary fall in the exogenous interest rate, a tentative initial conclusion is that flexibility in the intensity of use of the capital stock increases both the amplitude and persistence of the response to the change. A more definite, and somehow striking conclusion, is the initial "contractionary" effect on output of a fall in the interest rate. Once the incentive to economize on the use of "services per unit of capital" (*vis-à-vis* the use of "units of capital") in the production of total capital services is taken into consideration, the conclusion seems less striking.

In this paper the labor market is not modeled explicitly; nonetheless, it would be interesting to do so, in order to analyze how the variable utilization rate affects the response of labor hours to various shocks. The high volatility of total hours worked and the absence of a high correlation between hours worked and average labor productivity have constituted a puzzle within the real business cycle models. Ambler and Paquet [1994], attempted to solve this problem by assuming a stochastic rate of depreciation. The model presented here could also be suited to gain some insights concerning this puzzle.

The importance of a variable rate of services for capital as a propagation mechanism in business cycles was pointed out previously in Greenwood, Hercowitz and Huffman [1988]. In order to obtain similar results, one needs to model a temporary shock on the adjustment cost function $h(I)$. This has the effect of, say, reducing the marginal cost of investment and hence increasing the marginal efficiency of investment. Greenwood, Hercowitz and Huffman [1988] achieve this by way of a shock to the marginal productivity of investment.

DeJong, Ingram and Whiteman [1995] and DeJong, Ingram, Wen and Whiteman [1996] argue that a variable utilization rate of capital serves to characterize the properties of the US business cycle. It was pointed out here, that a variable utilization rate is not limited to being a propagation mechanism for certain *shocks*, but it also affects their persistence, magnitude and may even reverse their direction.

APPENDIX

Here, a proof of the effect of flexibility upon the saddle path is included.

Let D be the determinant of the system matrix in (8) when there is no flexibility, the negative eigenvalue, v, is then given by

$$v = \frac{-r - \sqrt{r^2 - 4D}}{2} \qquad \text{(A1)}$$

and the slope, m, of the saddle path is

$$m = \frac{-s^2 f''}{v - (r + \delta)} \qquad \text{(A2)}$$

On the other hand when flexibility in the choice of s is introduced, the negative eigenvalue and the slope of the saddle path are given respectively by

$$v^f = \frac{-r - \sqrt{r^2 - 4AD}}{2} \qquad \text{(A1')}$$

$$m^f = \frac{-s^2 f''}{\left(v^f / A \right) - (r + \delta)} \qquad \text{(A2')}$$

where

$$A \equiv \frac{p\delta''}{p\delta'' - kf''}$$

and D is the same determinant as above. It is not hard to check that, $|v| > |v^f|$ and $|m| > |m^f|$, this entails a lower speed of adjustment towards the steady state and a "flatter" saddle path when flexibility is introduced.

FOOTNOTES

[1] Notice that since adjustment costs depend on gross investment then, even in the long run steady state, the price of capital will be different from unity. Adjustment costs are not defined as a function of *I/k* since the fundamental results are not changed and the algebra is considerably more tedious.

[2] The assumption $\rho = r$ may be justified by assuming that the rest of the world is in equilibrium so that the real interest rate is equal to the rest of the world's consumers time preference rate. Individuals in this country are presumed to be no different than their foreign peers yielding $\rho = r$. There are certain problems associated with the other two possibilities. If $\rho > r$, individuals in this country are more impatient than those in the rest of the world; therefore, they mortgage all their capital and labor income in order to increase their present consumption driving their future consumption towards zero. In order to avoid this outcome, an international credit constraint on the country may be imposed. If $\rho < r$, the country is more patient than the rest of the world and will asymptotically accumulate all of the world's assets, once this happens, the "small country" assumption is untenable and the world real interest rate r cannot be assumed to be an exogenous constant. This issue is one of the problems of applying the Ramsey model to the open economy and it is throughly discussed in Barro and Sala-i-Martin [1995, pp. 103-108].

[3] If k is thought of as an inventory of final goods, increased depreciation during the adjustment amounts to faster liquidation of the inventory caused by the falling price of k.

Table 1

(β = 1.25)

b	α	δ*	p*	k*	λ_v	λ_f
1	0.75	0.21	1.273	0.649	-0.219	-0.318
2	0.75	0.21	1.387	0.460	-0.204	-0.296
3	0.75	0.21	1.465	0.369	-0.197	-0.287
4	0.75	0.21	1.527	0.313	-0.194	-0.282
5	0.75	0.21	1.577	0.275	-0.191	-0.278
1	0.3	0.21	1.233	0.554	-0.236	-0.477
2	0.3	0.21	1.391	0.466	-0.200	-0.408
3	0.3	0.21	1.518	0.411	-0.186	-0.380
4	0.3	0.21	1.627	0.373	-0.178	-0.365
5	0.3	0.21	1.722	0.344	-0.173	-0.355

Table 2

(β = 2)

b	α	δ*	p*	k*	λ_v	λ_f
1	0.75	0.06	1.619	5.155	-0.071	-0.082
2	0.75	0.06	1.803	3.346	-0.069	-0.079
3	0.75	0.06	1.926	2.572	-0.068	-0.078
4	0.75	0.06	2.020	2.125	-0.067	-0.077
5	0.75	0.06	2.097	1.829	-0.067	-0.076
1	0.3	0.06	1.227	1.888	-0.112	-0.151
2	0.3	0.06	1.382	1.592	-0.093	-0.127
3	0.3	0.06	1.507	1.408	-0.086	-0.117
4	0.3	0.06	1.613	1.277	-0.082	-0.112
5	0.3	0.06	1.707	1.178	-0.079	-0.109

Table 3

(β = 5)

b	α	δ*	p*	k*	λ_v	λ_f
1	0.75	0.023	2.562	34.713	-0.028	-0.029
2	0.75	0.023	2.900	21.121	-0.028	-0.029
3	0.75	0.023	3.123	15.726	-0.027	-0.028
4	0.75	0.023	3.292	12.734	-0.027	-0.028
5	0.75	0.023	3.430	10.802	-0.027	-0.028
1	0.3	0.023	1.219	4.867	-0.059	-0.065
2	0.3	0.023	1.370	4.117	-0.049	-0.054
3	0.3	0.023	1.492	3.646	-0.044	-0.049
4	0.3	0.023	1.596	3.311	-0.042	-0.047
5	0.3	0.023	1.688	3.057	-0.040	-0.045

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Figure 1

Basic Dynamics

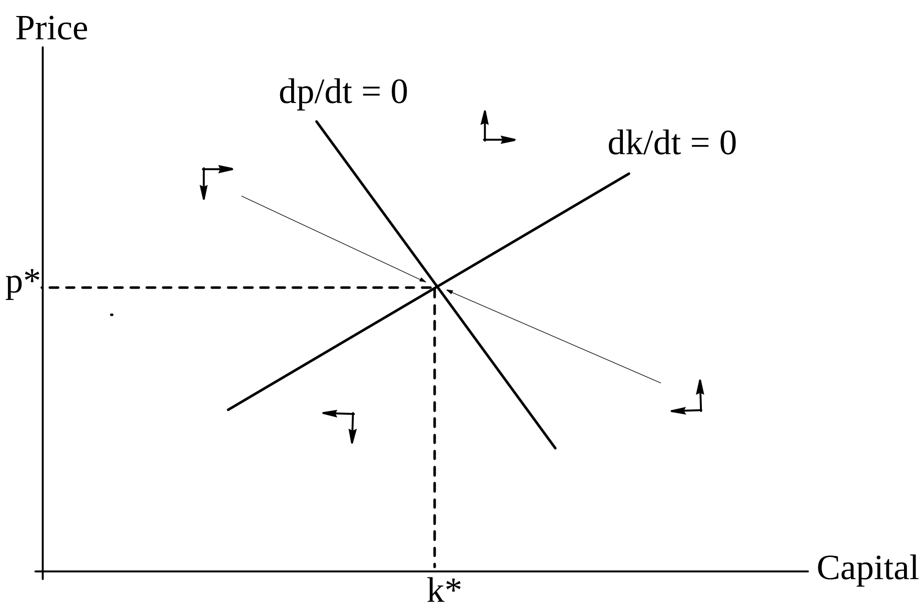

Figure 2
Price of Capital

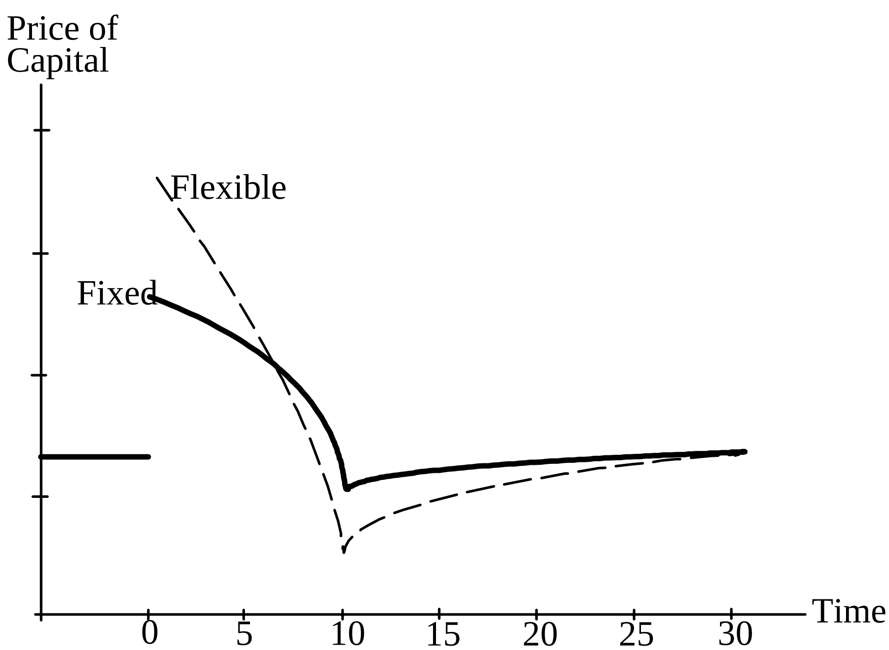

Figure 3
Capital Stock

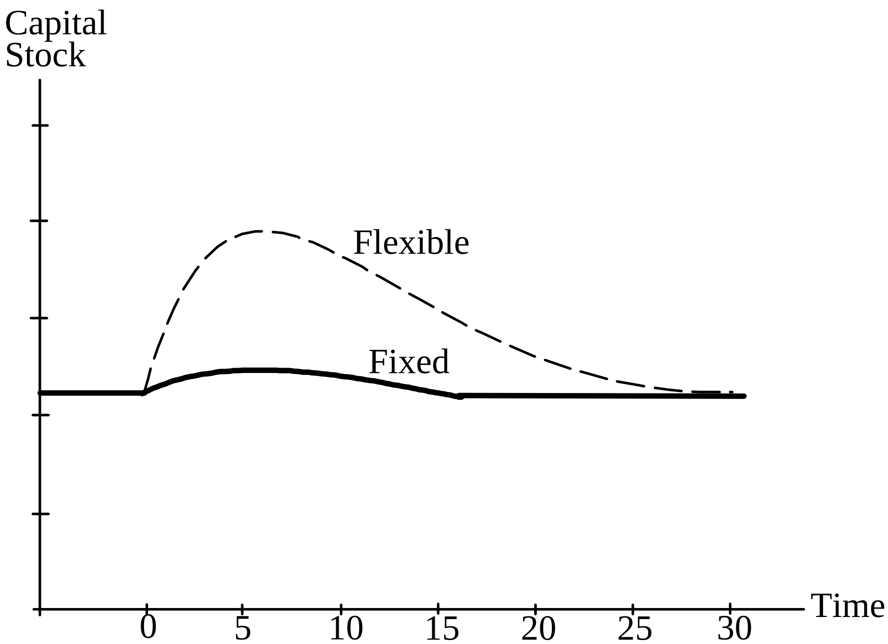

Figure 4
Utilization Rate

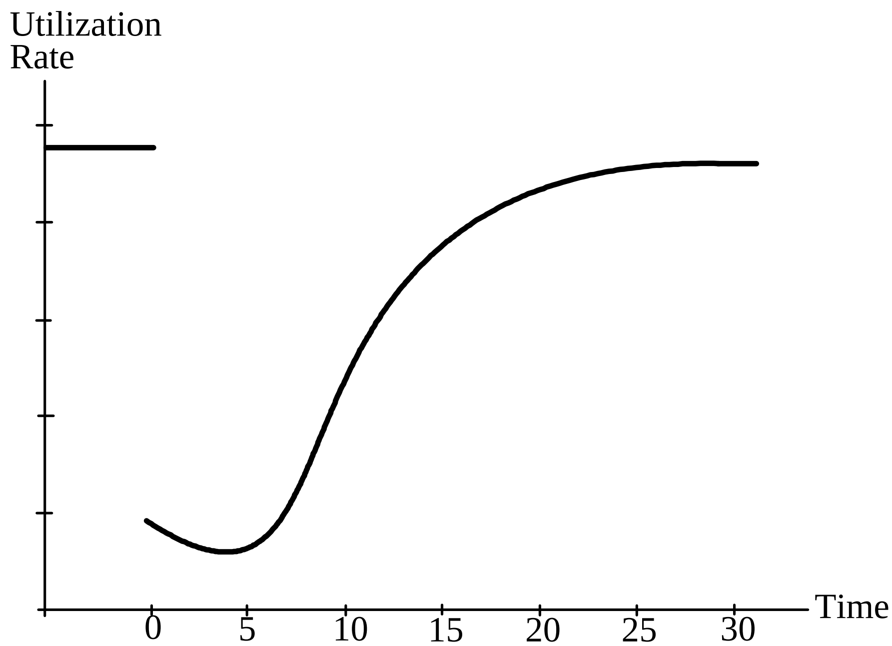

Figure 5
Output