Portfolio Choice and Corporate Financial Policy When There Are Tax-Intermediating Dealers*

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Abstract

Miller (1977) emphasized that optimal corporate financial policy depends on the tax rates facing the firm as well as the tax treatment accorded bond- and stock-holders. In equilibrium, firms will issue claims that are held by the full spectrum of investors, from tax-exempt institutional investors to heavily taxed individuals. However, dealers are tax neutral institutions that can buy corporate securities and, under some circumstances, reissue them as a financial package with different tax characteristics. This activity potentially severs the link between the securities the firm issues and the securities investors hold. In a continuous-time setting with dealers, I show that investors generally will hold derivatives, and possibly stock, in lieu of holding taxable bonds. Moreover, the marginal tax advantage to debt in this setting is the corporate tax rate.

Keywords: Taxation, Capital Structure, Corporate Finance

1 Introduction

Miller (1977) observed that both personal and corporate taxes should affect the corporate decision to finance with debt or equity. In equilibrium, the pre-tax returns on a security will reflect investor-level taxes: a security that

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is onerously taxed at the investor level will have a higher pre-tax required return than a security that is taxed lightly. Thus, the corporate decision to issue a particular security depends on whether the corporate tax deduction offsets the taxes paid by investors. In making a capital structure decision, therefore, Miller argued that firms implicitly take into account the investor-level taxes of the financial claims that firms issue.

Miller's analysis assumes that investors hold the securities that firms issue. However, this need not be true, and in practice it is often not true. Securities dealers commonly buy, sell, and hold positions in the financial instruments issued by firms and hedge the resulting risk by trading in other, related financial products such as options. Dealers and their subsidiaries sometimes even issue wholly new securities, based on, but different from, the claims issued by the firms. This kind of dealer activity cannot create or destroy aggregate risk, but it can alter the tax characteristics of the claims investors hold. The point of the paper is that this intermediation activity by dealers can break the link between the securities the firm issues and the securities that investors hold, and that if it occurs, this delinking fundamentally alters Miller's story.

Specifically, in a continuous-time setting based on the discrete time model in Auerbach and King (1983), I characterize portfolio equilibrium with and without dealers. I show that when income from derivatives is taxed as capital gains, investors will hold derivatives in lieu of debt and equity. The tax advantage to debt is then independent of personal taxes.

The argument in this paper hinges on two important considerations. First, there is a fundamental difference in the tax treatments of investors, firms, and dealers. For investors and firms, different forms of income, such as interest, capital gains, and dividends, receive different tax treatment. However, for dealers holding securities in inventory, all income from the securities is taxed as ordinary income, with gains and losses determined by marking to market. Consequently, when buying one set of securities and issuing another, dealers have the potential to alter the tax treatment of a given claim to economic income.

An obvious and well-known illustration of this point occurs when dealers make markets in options. A dealer who sells a call option, for example, will hedge and finance the resulting risk exposure by borrowing to buy shares of stock. For an investor, the three claims in this transaction — a call option, stock, and bonds — all have distinct tax rules, yet dealers are simply taxed on the net economic income resulting from the hedging transaction.

The second consideration is that issuers of securities often have considerable (but not arbitrary) latitude in determining how the cash flows from of a given stream of income are characterized for tax purposes. In Section 3.1 I discuss this in more detail but the general point is well-known. For example, Warren (1993) pointed out that investors can potentially alter the taxation of a given form of income by using a different investment vehicle to achieve the same pre-tax economic result. A synthetic bond created by buying stock, selling a call, and buying a put potentially receives different tax treatment than a bond. Even if a regulation recognizes that the combination of stock, call and put is equivalent to a bond and that therefore the income from the position should be taxed as interest, it may be possible to modify the transaction slightly (for example by setting different strikes on the call and put) to avoid this regulation, albeit at the cost of no longer having an exact bond-equivalent.

To see the implications of these points for firm capital structure decisions, consider Miller's original argument demonstrating, in a world of certainty, that there is an aggregate equilibrium capital structure. In the simplest Miller world, firms can issue untaxed equity or debt with interest that is deductible for firms at the rate τ and taxable to investors at the rate u. Suppose there are investors with tax rates less than and greater than τ. Investors will hold whichever security has, for them, a greater after-tax return. Firms in this world will, in the aggregate, issue debt until the marginal investor has a tax rate of $u^* = \tau$. Given a pre-tax return on equity of α, the pre-tax return on debt will in equilibrium be $\alpha/(1-\tau)$. Investors with $u > \tau$ hold equity, others hold debt, and firms are indifferent about issuing an additional dollar of either debt or equity.

Suppose we introduce into this setting dealers who have lower transaction costs than other traders and who can buy one claim and issue a different claim, effectively recharacterizing the nature of income for tax purposes. The Miller equilibrium then breaks. Dealers will buy corporate debt paying $\alpha/(1-\tau)$, and issue corporate equity paying the return α. Dealers stop intermediating only when the *pre-tax* returns on equity and debt are equal, at which point firms will exclusively issue debt, or at least issue debt until they have exhausted taxable income.

Of course, dealers do not in practice have the ability to arbitrarily recharacterize income. The tax rules and authorities examine characteristics of securities to guide classifications. The recent adoption of new rules in the U.S. governing contingent debt illustrates that tax law is dynamic. Still, as the example of an option dealer illustrates, recharacterization of income does occur and, depending on the context, can be relatively non-controversial. Moreover, as I discuss in Section 3.1, it is sometimes possible for an issuer to select the desired tax treatment for a given stream of income. Generally, the question is: what is the appropriate tax treatment for a security with a payoff that is a non-linear function of a stock price? An option is an example of a claim that is equivalent to a position in a stock and bond, yet which is accorded capital gains treatment. We can think of investors implementing tax reduction strategies by holding options.[1]

The idea in this paper is related to the general idea that firms can increase after-tax income by issuing claims other than debt and equity. Titman (1985) and Warren (2000) both discuss a transaction in which firms can essentially issue tax deductible equity.[2] In this transaction, firms issue bonds and enter into short forward contracts on equity, obligating the firm at the bond maturity date to sell equity in exchange for the bond's terminal repayment value. At the maturity date, this transaction generates the same number of shares outstanding and cash position as if the firm had currently issued equity and invested the proceeds in debt, yet the interest on the bond generates a tax deduction. The forward contract is untaxed for the firm. The net effect is that the firm has untaxed income due to the forward contract. Although this and similar transactions are recognized as problematic by tax authorities, their tax status remains unclear.

It is worth emphasizing that dealers can accomplish tax intermediation in a way that firms themselves cannot. It is difficult for firms to simultaneously obtain financing tax deductions and flexibly accomdate tax clientele demands. Firms in some circumstances have considerable latitude to characterize the character of securities they issue, but as Kleinbard and Nijenhuis (1996) note, it is important that all parties to a transaction agree on a common characterization for income tax purposes. Characterizing a payment as interest for the issuer generally imposes interest taxation on the holder. In some circumstances, dealers have the unique ability to take two sides of a position where the parties have elected different tax characterizations.[3]

I make a number of simplifying assumptions with respect to taxes. I do

[1] Neuberger and Hodges (2002) discuss the value of options in a setting with stochastic volatility. In this paper, volatility is non-stochastic.

[2] McDonald (forthcoming) also discusses this transaction and generalizes the discussion to show how issuing options can also create non-taxable implicit debt holdings.

[3] A recent skirmish over hedge-fund investment vehicles illustrates limitations on dealer-facilitated arbitrage. Hedge-funds are generally organized as partnerships with income passed-through to investors. Dealers in this case created specific investment vehicles to alter the tax treatment of the hedge fund investment, converting partnership income to capital gains income. Constructive ownership rules were added to the tax code to defeat these transactions. For a specific discussion of this issue, as well as a more general discussion of factors limiting the ability of the tax authorities to craft rules constraining arbitrage activity, see Schizer (2001).

not consider the kinds of shareholder-level tax minimization strategies that are considered by Miller and Scholes (1978) and I assume that investors are completely taxable. The growth of tax-deferred accounts has made this less true over time. In particular the ability of investors to hold bonds in a tax-deferred account (see, e.g. Dammon, Spatt, and Zhang, forthcoming) should in and of itself permit firms to increase leverage by reducing shareholder-level taxation. However in this paper I don't consider the effects of retirement accounts. I also assume that capital gains taxes are paid on accrual, thus ignoring the capital gains deferral strategies that are undeniably important in practice.

A number of other papers have dealt with tax clienteles, and one of the issues that generally arises is the need for short-sale restrictions to prevent high-tax-bracket investors and low-tax-bracket investors from taking offsetting long and short positions to trade with each other, and thereby either creating a money pump or equating their marginal tax rates. The need for a short-sale constraint is particularly acute when investors are risk-neutral, but the investors in this paper are risk-averse.

Section 2 discusses prior related research and Section 3 discusses the tax issues. In Sections 4 and 5 I characterize equilibrium with and without dealers. Section 6 presents some examples to illustrate practical problems with implementing the equilibrium I discuss. Section 7 concludes.

2 Theory and Evidence on Personal Taxes and Capital Structure

Theory suggests that firms, when making financial decisions, should take into account the taxation of counterparties, such as bondholders and stockholders. Counterparty taxation is potentially quite important since it affects the net tax benefit of issuing debt. In practice it is often assumed in capital budgeting calculations that a firm issuing debt benefits from interest tax deductions, as in Modigliani and Miller (1958). However, in the model of Miller (1977), debt has no net tax advantage and there is no tax benefit to leverage. Because of this use in capital budgeting, it is potentially quite important to have a sense of the magnitude of the net tax advantage to debt.

Many discussions of the tax benefits of leverage, e.g., Miller (1977) and DeAngelo and Masulis (1980), assume, in effect, that there is risk-free equity: a security with the risk of a bond that is taxed like stock.[4] If risk-free

[4]More generally, as Auerbach and King (1983) discuss, investors can separate the tax

equity exists, high tax bracket investors will hold risk-free equity rather than debt. By comparing the yields on the two securities, it would then be possible to identify the marginal investor and estimate the net tax disadvantage to debt. Thus, in theory it could be possible to distinguish these views of the value of debt by examining security returns.[5] However, risk-free equity does not exist and this kind of calculation is empirically infeasible except in narrow circumstances. For an example, see Engel, Erickson, and Maydew (1999), who examine exchanges of preferred stocks for trust preferred, which is taxed like debt.

Although the magnitude of the tax advantage to debt is unknown, Graham (1999) provides evidence that firm-specific tax rate measures, incorporating a measure of shareholder-level taxation, are of modest help in explaining cross-sectional leverage differences.

In a recent survey of the literature, Graham (2003) concludes "The truth is that we know very little about the identity or tax-status of the marginal investor(s) between any two sets of securities, and deducing this information is difficult."

3 Tax Issues

The U.S. tax code, like that of many other countries, distinguishes among different forms of income when specifying tax rates and rules. Wages, dividends, interest, and capital gains are distinct categories of income in the U.S. There are two forces working in tandem that undermine these distinctions, however. First is the flexibility that the issuers of financial claims have to characterize income in different ways for tax purposes. Second is the tax-neutral character of dealers, which makes them uniquely suited to satisfy demands for securities with particular tax characteristics. In this section I discuss these two tax issues, which are central in this paper.

3.1 The Tax Treatment of Financial Income

Distinctions in the tax law between forms of income from financial assets are often not clear, either in theory or practice. Warren (1993) uses options to provide an illustration of the basic conceptual problem. There are

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and risk-bearing dimensions of portfolio choice.

[5] In practice, even questions that one would expect to have well-defined answers do not. See for example the discussions of the ex-dividend-day effect in Boyd and Jagannathan (1994) and Elton, Gruber, and Blake (2003).

distinct rules for taxing the income from bonds, stocks, and options. However, because of put-call parity, options can be combined with a bond to create a synthetic stock, stocks and options can be combined to create a synthetic bond, etc. Even the question of whether a financial claim is or is not debt may not be clear, since the bonds of highly-leveraged firms exhibit characteristics of equity.

Convertible bonds provide another example of elective tax treatment. With a standard convertible bond, the coupon on the bond will be less than that on an equivalent non-convertible bond because of the conversion feature. The holder of the bond is taxed on the actual coupon, which is also deductible for the issuer, and the gain on conversion is a capital gain. If the firm instead issues an ordinary bond and a warrant, the larger coupon is taxable to the holder and deductible to the issuer. Recent regulations governing so-called "contingent debt" (see Kleinbard, Nijenhuis, and McRae, 2002) provide a third alternative for taxing a convertible.[6] Firms can add a "non-incidental" and "non-remote" contingency to the interest payments on the bond. The bond is then taxable under the contingent debt regulations, in which case the firm can deduct from income the coupon that *would* be paid on an otherwise equivalent noncontingent bond. In addition, gains from conversion on such a bond are taxed as ordinary income rather than capital gains, with the same rules applying to the holder of the bond. There are additional rules governing adjustments at maturity or conversion of the bond; the point of this discussion is that the firm has considerable latitude to choose the tax treatment applicable to similar cash flows.

Modern financial contracts, such as equity-linked notes, muddy the tax issues even further. Schizer (2000a) discusses the example of DECS, which are notes that make a periodic interest-like payment and at expiration mandatorily convert into stock, with a conversion ratio that depends on the expiration stock price.[7] According to Schizer "there is no clear authority indicating the tax treatment for DECS" and he offers three plausible ways to tax such a claim: as contingent debt, as a prepaid forward contract, and as debt plus an ordinary forward contract. Iyer (1998) provides a similar perspective for PEPS (another name for DECS). This report by an investment bank defines "equity PEPS", with coupons taxed like dividends and the conversion into shares of the issuing company tax-free; "debt PEPS", with the coupon taxed like interest and the conversion into shares tax-free;

[6] See also Shefter (1996) and Schizer (2000b).

[7] McDonald (2003, Chapter 15) discusses examples of equity-linked notes, discusses their pricing, and covers some motivations for firms to issue them.

and “trust PEPS”, taxed like bonds and a forward contract.

The ability to classify income to obtain a desired tax treatment is not arbitrary; the tax authorities attempt to guide characterizations and there is a strong presumption that a claim that looks like a bond will be taxed like a bond. Section 1258 of the Internal Revenue Code, for example, states that the gain on a transaction is ordinary income rather than capital gains when different part of the transaction were entered into simultaneously and “substantially all of the taxpayer’s expected return ... is attributable to the time value of the taxpayer’s net investment”. In other words, if a transaction has the effect of creating a synthetic bond, it is taxed like a bond.

As was originally shown by Black and Scholes (1973), given certain assumptions, an option has a return equivalent to some (dynamically varying) position in the underlying asset and a risk-free bond. Nevertheless, option gains are taxed as capital gains. Options thus provide a conceptual challenge for the tax code’s attempt to distinguish between interest income and capital gains income.

3.2 The Tax Treatment of Dealers

The U.S. tax code has different rules for assessing income tax on individuals and businesses, including securities dealers. In this section I discuss the differences that will be important in this paper.

When assessing tax on an asset, the U.S. tax code draws a distinction between assets held for investment purposes and those held as part of the normal course of a trade or business. For an investor, the price change of an asset is treated as capital gain or loss and is taxed only when the asset is sold.[8] For dealers, by contrast, since 1993, asset positions have been marked-to-market and taxed annually, with the gain or loss treated as ordinary income. (Prior to that time dealers received ordinary income treatement but assessed the change in value using a “lower of cost or market rule”.) In effect, for dealers, assets positions are treated as inventory held by a business. The ability of dealers to exploit their tax neutrality and serve as “tax accomodation counterparties” is discussed in Schizer (2001).

If dealers are the market makers in the sense of providing the best bid and ask for a security (as would be expected if they have the lowest transaction costs and the readiest access to capital) then dealer tax treatment should be

[8]There are numerous special rules and qualifications to a statement like this; for example, as recognized in Section 1259 of the U.S. tax code, it is not always obvious what it means to sell an asset. The purpose of this discussion is to contrast the tax treatments of investors and dealers.

reflected in market prices. The impact of taxes on derivative prices was studied by Scholes (1976), Cornell and French (1983), and Cox and Rubinstein (1985, pp. 271-274), who showed how prices depend upon taxes when capital gains, dividends, and interest are taxed at different rates. In these models, an investor who is taxed at same rate on all forms of income—such as a dealer—will price a derivative the same as a tax-exempt investor. This is why dealers are effective at accomodating demands of taxed counterparties.

4 Equilibrium Without Dealers

In this section I present a continuous-time version of the Auerbach and King (1983) model and restate many of their results. The purpose is to characterize equilibrium in the absence of dealer tax intermediaries. In the next section I introduce dealers into this setting.

There are n investors in different tax brackets and m firms that finance investment by issuing default-free debt and equity. The subscripts i and j denote investors and firms, so that firm j's debt and equity are B_j and S_j. Firm j has capital stock K_j with which it produces after-tax operating income of dK_j. I assume that leverage and the capital stock can be instantaneously adjusted, so that equilibrium in the market for physical capital entails that $K_j = B_j + S_j$, i.e., Tobin's $q = 1$. I suppress the time subscript on variables when it is not needed.

The purpose of this model is to characterize the valuation effects of debt, taking as given the distribution of tax characteristics across heterogeneous investors. I do not permit firms to issue securities other than debt and equity. This assumption is in some sense at the heart of the paper and deserves discussion, especially since firms could possibly issue a menu of tailored securities to exploit clientele effects. Firms do not have a comparative advantage in designing and issuing such tailored financial products. Moreover, products that benefit investors may by suboptimal for firms. In designing a capital structure, firms have multiple goals: minimization of agency costs, adverse selection costs, financial distress costs, and transaction costs. In theory, firms also minimize joint shareholder-level and firm-level taxes. Simultaneously attending to these goals may require trade-offs. Tax-intermediating dealers may permit firms to ignore shareholder-level taxes and focus on other issues.

4.1 Firms

Given physical capital of K_j, firm j produces after-tax operating cash flow of dK_j, which follows the process[9]

$$dK_j = X_j K_j dt + \sigma_j K_j dZ_j \tag{1}$$

I assume that all cash flows are paid as dividends, share repurchases, or interest, so that firms do not invest in financial assets and payments to equity holders are the marginal source of funds. The firm is taxed at the rate τ. If the firm has B_j in debt outsanding, it pays the pre-tax flow of interest $rB_j dt$ to bondholders and reduces shareholder payments by $-r(1-\tau)B_j dt$. I assume that debt is risk-free.[10] I assume the pre-tax dividend yield is fixed at δ_j. The firm then repurchases $dK_j - (\delta_j S_j + r(1-\tau)B)dt$, which is therefore paid to shareholders as capital gains. Let the $m \times 1$ column matrix denote the stacked expected capital gain for all firms:

$$\alpha = \mathbf{S}^{-1}[X - S\delta - r(1-\tau)B] \tag{2}$$

The after-corporate-tax cash flow covariance matrix for all m firms is denoted by Ω, where the $\{i, j\}$ entry is

$$\frac{1}{dt} E(\sigma_j dZ_j \sigma_k dZ_k) = \sigma_{jk}$$

The pre-tax capital gain on the stock is

$$dS_j = \alpha_j S_j dt + v_j S_j dZ_j \tag{3}$$

where α_j is the expected pre-tax capital gain on the stock and v_j is the stock volatility. Since debt is risk-free, the volatility of the stock return is $v = \sigma_j X_j / S_j$.

Let $\mathbf{V}$ denote the variance-covariance matrix of equity returns and let

$$\mathbf{S} = \begin{pmatrix} S_1 & 0 & \dots & 0 \\ 0 & S_2 & \dots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & S_m \end{pmatrix} \qquad \mathbf{K} = \begin{pmatrix} K_1 & 0 & \dots & 0 \\ 0 & K_2 & \dots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & K_m \end{pmatrix}$$

We then have

$$\mathbf{V} = \mathbf{S}^{-1}\mathbf{K}\boldsymbol{\Omega}\mathbf{K}\mathbf{S}^{-1} \tag{4}$$

[9] I intend X to include all items affecting firm cash flow other than interest payments.

[10] This is a consistent assumption if firms readjust capital structure as stock prices change. Zechner (1990) shows that interesting clientele effects arise among bondholders when firms can default on debt.

4.2 Investors

Investor i selects consumption and portfolio weights to maximize the utility of lifetime consumption, $J(W, t)$,

$$J(W,t) = \mathrm{E}_t \int_t^{\infty} e^{-\rho(s-t)} U(C_s) ds \tag{5}$$

where ρ is the rate of time preference. Investors pay taxes at the rate c_i on capital gains, d_i on dividends, and u_i on interest, and allocate wealth, W_i, among risk-free bonds and shares of stock, with ω_{ij} denoting the fraction of investor i's wealth invested in firm j. Investor i's vector of portfolio demands for the m stocks is ω_i, the vector of expected capital gains is α, and the vector of dividend yields is δ. The vector of m ones is $\boldsymbol{\iota}$. The process for wealth for the i^{th} investor is given by

$$\begin{aligned} dW_i = -C_{i,t}dt + W\big\{\omega'\big[\alpha(1-c_i) + \delta(1-d_i) \\ - r\boldsymbol{\iota}(1-u_i)\big]dt + (1-c_i)\sum_{j=1}^{m} \omega_j v_j dZ_j + r(1-u_i)dt\big\} \end{aligned} \tag{6}$$

The Bellman equation for this problem is

$$\max_{C_t,\omega} \left[e^{-\rho t} U(C_t) + J_W E(dW) + \frac{1}{2} J_{WW} E(dW^2) + J_t + \phi\omega + \psi(1 - \boldsymbol{\iota}'\omega) \right] \tag{7}$$

The Lagrange multipliers ϕ and ψ are for non-negativity constraints on shareholdings and bonds.

The first-order condition for ω is

$$\begin{aligned} \left[\alpha(1-c_i) - r(1-u_i)\boldsymbol{\iota} + \delta(1-d_i)\right] J_W \\ + (1-c_i)^2 J_{WW} W^2 \mathbf{V}\omega + \phi - \psi\boldsymbol{\iota} = 0 \end{aligned} \tag{8}$$

where ϕ and ψ are the Lagrange multipliers for the constraints $\omega'\boldsymbol{\iota} \geq 0$ (no short-selling of the aggregate share position) and $\omega'\boldsymbol{\iota} \leq 1$ (no borrowing). Solving for ω gives

$$\omega^i = \left(-\frac{J_W}{(1-c_i) W J_{WW}} \right) \mathbf{V}^{-1} \left(\alpha - r\boldsymbol{\iota}\frac{1-u_i}{1-c_i} + \delta\frac{1-d_i}{1-c_i} + \frac{\phi - \psi\boldsymbol{\iota}}{1-c_i} \right) \tag{9}$$

4.3 Equilibrium Without Dealers

To obtain equilibrium I aggregate individual investor demands and equate aggregate demand to supply. Set $S\boldsymbol{\iota} = \sum_i \omega^i W_i$ and let $A_i = -J^i_W/(1-c_i)J^i_{WW}$. We have

$$\sum_i \omega^i W_i = \mathbf{S}\boldsymbol{\iota} = \sum_i A_i \mathbf{V}^{-1}\left(\alpha - r\boldsymbol{\iota}\frac{1-u_i}{1-c_i} + \delta\frac{1-d_i}{1-c_i} + \frac{\phi-\psi}{1-c_i}\right) \tag{10}$$

Using equation (2) to substitute for α, we have

$$\mathbf{S}\boldsymbol{\iota} = \sum_i A_i \mathbf{V}^{-1}\Bigg(\mathbf{S}^{-1}[X - \mathbf{S}\delta - r(1-\tau)B] - r\boldsymbol{\iota}\frac{1-u_i}{1-c_i} + \delta\frac{1-d_i}{1-c_i} + \frac{\phi-\psi\boldsymbol{\iota}}{1-c_i}\Bigg) \tag{11}$$

Since $\mathbf{V} = \mathbf{S}^{-1}\mathbf{K}\Omega\mathbf{K}\mathbf{S}^{-1}$, we can solve for the market value of the firm, $\mathbf{S}+\mathbf{B}$, in terms of the cash flows of the firm:

$$\mathbf{S}\boldsymbol{\iota} = \sum_i A_i \mathbf{S}[\mathbf{K}\Omega\mathbf{K}]^{-1}\mathbf{S}\Bigg(\mathbf{S}^{-1}[X - \mathbf{S}\delta - r(1-\tau)B\boldsymbol{\iota}] - r\boldsymbol{\iota}\frac{1-u_i}{1-c_i} + \delta\frac{1-d_i}{1-c_i}\Bigg) \tag{12}$$

Let $A_d = \sum_i A_i(1-d_i)/(1-c_i)$, $A_u = \sum_i A_i(1-u_i)/(1-c_i)$, and $A = \sum_i A_i$. Assuming that short-sale constraints are not binding and rewriting gives

$$\mathbf{S}\alpha + \mathbf{S}\delta\frac{A_d}{A} = r\frac{A_u}{A}K\boldsymbol{\iota} + \frac{1}{A}\mathbf{K}\Omega\mathbf{K}\boldsymbol{\iota} \tag{13}$$

This can also be written

$$X\boldsymbol{\iota} + rB\boldsymbol{\iota}\left[\frac{A_u}{A} - (1-\tau)\right] + \mathbf{S}\delta\left(\frac{A_d}{A} - 1\right) = r\frac{A_u}{A}\mathbf{K}\boldsymbol{\iota} + \frac{1}{A}\mathbf{K}\Omega\mathbf{K}\boldsymbol{\iota} \tag{14}$$

To interpret this expression, suppose there are no taxes. The left hand side is the total expected dollar return to equity and debtholders, X. (Interest is paid from X, and hence is implicitly added and subtracted on the left-hand side of equation (14)). The right-hand side is the risk-free return on total assets plus a risk premium which for asset j is the covariance of the asset j cash flow with that of all other firm cash flows, divided by the average absolute risk tolerance of the economy. The last term on the left-hand side vanishes when taxes are zero.

When there are taxes, equation (14) gives the expected return on the firm as a function of the distribution of tax rates and risk tolerances across investors. The term multiplying rB is the net tax advantage to debt. In the special case in which all investors are in the same tax bracket, the term in brackets divided by A reduces to $(1-u)/(1-c)-(1-\tau)$, which is Miller's expression for the tax advantage to debt. The term containing X is the payout in the absence of debt, and the last term on the left-hand side is the increase in the payout from debt. The term multiplying $\mathbf{S}\delta$ reflects the tax disadvantage to dividends.

Equation (14) illustrates a point made by Auerbach and King (1983), which is that the tax advantage to debt depends on a weighted average of investor tax rates. Thus, there is not necessarily a "Miller equilibrium" in this setting. Given equation (14), a firm maximizing value would either be all-debt or all-equity financed, depending on the sign of the term multiplying rB.

We can examine the extent to which investors will tax-tilt their portfolios. Substituting X from equation (14), into equation (9), we obtain

$$\omega^i W_i = A_i \mathbf{V}^{-1}\mathbf{S}^{-1}\left\{\frac{1}{A}\mathbf{K\Omega K} + r\mathbf{K}\boldsymbol{\iota}\left[\frac{A_u}{A} - \frac{1-u_i}{1-c_i}\right] + \mathbf{S}\delta\left[\frac{1-d_i}{1-c_i} - \frac{A_d}{A}\right] + rB\boldsymbol{\iota}\left[\frac{A_d}{A} - \frac{1-d_i}{1-c_i}\right]\right\} \quad (15)$$

If there are no taxes, or if an investor has ratios of tax rates equal to the weighted average ratios of all investors, then the investor holds the market portfolio. Otherwise, taxes imply that the investor tilts the relative holdings of individual stocks based on the particular firm's leverage (the term in equation (15) containing B) or dividends (the term in equation (15) containing X).

4.4 Unanimity

The fact that shareholders hold different portfolios suggests that they will disagree about financial policy decisions. Auerbach and King (1983) show that tax heterogeneity induces diagreement.

Differentiating an investor's instantaneous change in utility with respect to the debt level of the k^{th} firm, we have

$$\frac{\partial E(dJ)}{\partial B_k} = \frac{\partial(E(dW))}{\partial B_k} J_W + \frac{1}{2}\frac{\partial(dW)^2}{\partial B_k} J_{WW} \tag{16}$$
$$= \left[\frac{\partial \alpha_k}{\partial B_k}(1-c_i) + \frac{\partial \delta_k}{\partial B_k}(1-d_i)\right]\omega_k J_W W + \frac{1}{2}\frac{\partial \omega' \mathbf{V}\omega}{\partial B_k} W^2 J_{WW}(1-c_i)^2$$

Since I assume δ is fixed, $\partial\delta/\partial B_k = 0$. The idea behind this calculation is to use the individual portfolio first-order condition, equation (9), and the aggregate equilibrium condition, equation (14), to evaluate the change in utility. The definition of α is

$$\alpha = \mathbf{S}^{-1}[X - \mathbf{S}\delta - r(1-\tau)B]$$

Substituting for X from equation (14), we obtain

$$\alpha = r\frac{A_u}{A}\boldsymbol{\iota} + \frac{1}{A}\mathbf{S}^{-1}\mathbf{K\Omega K}\boldsymbol{\iota} - \delta\frac{A_d}{A} \tag{17}$$

Since the investor optimally chooses ω_k, differentials with respect to ω_k drop out. Also, the assumption that $q = 1$ implies that $S_k = K_k - B_k$, so that

$$\frac{\partial S_k}{\partial B_k} = -1$$

From the definition of α_k, and taking into account the induced change in $\mathbf{S}$), $\partial\alpha_k/\partial B_k$ is

$$\frac{\partial \alpha_k}{\partial B_k} = \frac{1}{S_k}\left[\alpha + \delta - r(1-\tau)\right] \tag{18}$$

From equation (4) we can write the variance of the change in wealth as

$$\frac{1}{2}J_{WW}W^2\omega'\mathbf{V}\omega = \frac{1}{2}J_{WW}W^2\sum_i\sum_j \omega_i\omega_j(S_iS_j)^{-1}K_iK_j\sigma_{ij}$$

Evaluating the right-hand side of equation (16), we obtain

$$\omega_k W J_W(1-c_i)\frac{1}{S_k}\left[\alpha_k + \delta_k - r(1-\tau)\right]$$
$$+ \frac{1}{S_k^2}\omega_k(1-c_i)^2 J_{WW}W^2\sum_i \omega_i S_i^{-1}K_iK_k\sigma_{ik} \tag{19}$$

The last term can be rewritten as

$$\frac{1}{S_k^2}\omega_k K_k(1-c_i)^2 J_{WW}W^2\Omega_k K S^{-1}\omega$$

where Ω_k is the k^{th} row of Ω. Using equation (9) this can be further rewritten as

$$\frac{1}{S_k^2}\omega_k(1-c_i)^2 K_k J_{WW} W^2 \Omega_k K \mathbf{S}^{-1} \times \\ \left(-\frac{J_W}{(1-c_i)WJ_{WW}}\right)\mathbf{V}^{-1}\left(\alpha - r\boldsymbol{\iota}\frac{1-u_i}{1-c_i} + \delta\frac{1-d_i}{1-c_i} + \frac{\lambda_1-\lambda_0}{1-c_i}\right)$$

Thus, combining terms and using the definition of $\mathbf{V}$, equation (19) can be rewritten as

$$\omega_k W J_W(1-c_i)\frac{1}{S_k^2}\Bigg\{S_k\left[\alpha_k+\delta_k-r(1-\tau)\right] \\ -\Omega_k K_k \mathbf{K}\mathbf{S}^{-1}\mathbf{S}[\mathbf{K}\boldsymbol{\Omega}\mathbf{K}]^{-1}\mathbf{S}\left(\alpha - r\boldsymbol{\iota}\frac{1-u_i}{1-c_i} + \delta\frac{1-d_i}{1-c_i} + \frac{\lambda_1-\lambda_0}{1-c_i}\right)\Bigg\} \quad (20)$$

Use equation (17) to substitute for α_k and α. After simplifying, we obtain

$$\omega_k W J_W(1-c_i)\frac{1}{S_k^2}\Bigg\{S_k r\left[\frac{A_u}{A}-(1-\tau)\right] + S_k\delta_k\left[1-\frac{A_d}{A}\right] \\ -\Omega_k K_k \mathbf{K}[\mathbf{K}\Omega\mathbf{K}]^{-1}\left[\mathbf{S}r\boldsymbol{\iota}\left(\frac{A_u}{A}-\frac{1-u_i}{1-c_i}\right)+\mathbf{S}\delta\left(\frac{1-d_i}{1-c_i}-\frac{A_d}{A}\right)\right]\Bigg\} \quad (21)$$

Equation (21) shows that investors can disagree about a financial policy change because their tax rates differ from the weighted average tax rates for all investors and because of the Miller condition for debt being tax advantaged. The first two terms within curly brackets pertain to the investor's evaluation at the level of the firm; the remaining terms show that the investor's evaluation depends on the correlation of the firms with the market.

It is revealing to examine equation (21) when X_k is uncorrelated with cash flows of other firms. In that case the investor favors an increase in debt when the following expression is positive:

$$\omega_k W J_W(1-c_i)\frac{1}{S_k}\left(r\left[\frac{1-u_i}{1-c_i}-(1-\tau)\right]+\delta_k\left[\frac{1-d_i}{1-c_i}-\frac{A_d}{A}\right]\right)$$

When the firm pays no dividends, this is just the Miller condition using individual tax rates. When dividends are positive, a change in the debt level changes the fraction of shareholder income taxed at the dividend rate, so shareholders care about their dividend tax rate relative to the average for shareholders.

5 Equilibrium With Dealers

In this section I introduce dealers in addition to investors and firms. As discussed in Section 3.2, dealers are tax-neutral conduits with a technology that permits them to buy the debt and equity issued by firms and issue claims with different risk and tax characteristics.

Dealers can step between firms and investors, creating securities that have tax and risk characteristics different than the securities issued by the firm. In this section I suppose that dealers can create securities that, for tax purposes, resemble non-dividend-paying stocks for which the entire return is taxed as capital gains. I will discuss the realism of this assumption and qualify it in a later section, but at this point my goal is to see how this ability affects the equilibrium.

5.1 Structured Claims vs. the Underlying Stock

I assume that dealers issue a structured claim based on the stock of a firm and that this claim has a continuous payoff that can be replicated in standard fashion. At a point in time the price and risk of the claim can be described by its replicating portfolio. Let Q_k denote the price of a claim based on the k^{th} stock price, and let Δ_k and D_k denote the stock and bond position that replicates the note. The price of the claim is

$$Q_k(S_k) = \Delta_k S_k + D_k \tag{22}$$

The claim will have a return perfectly correlated with that of the stock. The elasticity of the claim with respect to the stock, η_k, is defined as

$$\eta_k = \frac{S_k \Delta_k}{Q_k(S_k)} \tag{23}$$

We can think of the claim as containing implicit shares, $S_k\Delta_k$, and an implicit bond worth D_k. Given the price and the elasticity, the definition of elasticity implies that the dollar value of the implicit shares is

$$\eta_k Q_k = S_k \Delta_k \tag{24}$$

The implicit debt in the claim is

$$D_k = Q_k(1 - \eta_k)$$

The volatility of the claim relative to the underlying stock will also depend on the elasticity:

$$v_{\text{claim}} = \frac{S_k}{Q_k}\frac{\partial Q_k}{\partial S_k} v_k = \frac{S_k \Delta_k}{Q_k} v_k = \eta_k v_k \tag{25}$$

For the moment I assume that these claims can be created so that all payments are taxed as capital gains. I will later discuss this assumption along with structuring issues more generally.

For future reference, notice that the elasticty of firm value with respect to the stock price is just the equity to asset ratio, $S/K = S/(S + B)$.

To analyze the structured claims, let $\boldsymbol{\eta}$ denote the $m \times m$ matrix with the elasticity, η_j, on the diagonal and zeros elsewhere. The payoff on stock j and claim j are perfectly correlated and thus, in order to not give rise to dealer arbitrage, must have the same *pre-tax* Sharpe ratio.[11] Letting α^* denote the pre-tax expected capital gain on the claim and δ^* the pre-tax dividend on the note, we have

$$\frac{\alpha^* + \delta^* - r}{\eta v} = \frac{\alpha + \delta - r}{v} \tag{26}$$

This equation implies that

$$\alpha^* = r + \eta(\alpha + \delta - r) - \delta^* \tag{27}$$

Let y be the vector of portfolio weights associated with the claims and f the tax rate on dividend income on the claims. The expected change in wealth is

$$\begin{aligned} E(dW) = W \big\{ \big[\alpha(1 - c_i) + \delta(1 - d_i) - r\boldsymbol{\iota}(1 - u_i)\big]\, \omega \\ + \left[\alpha^*(1 - c_i) + \delta^*(1 - d_i) - r\boldsymbol{\iota}(1 - u_i)\right] y \big\} \end{aligned} \tag{28}$$

The instantaneous covariance matrix for the m stocks and the m claims based on the stocks is

$$\mathbf{R} = \begin{pmatrix} \mathbf{V} & \boldsymbol{V\eta} \\ \boldsymbol{V\eta} & \boldsymbol{\eta V \eta} \end{pmatrix} \tag{29}$$

In order to rule out investors undertaking tax arbitrage positions against dealers, I assume that investors cannot take offsetting short and long positions in the claim and the underlying stock. This seems a plausible restriction for two reasons. First, the tax treatment of such a transaction is uncertain: if the resulting hedged position behaves like a bond, it might be taxed in its entirety like a bond, in which case the investor ends up making zero after-tax profits, like the dealer. Second and perhaps more importantly, the transactions costs would be high for investors compared to dealers, particularly if the transaction entailed dynamically changing hedge positions.

[11] The result that perfectly correlated assets must have the same Sharpe ratio is due to Merton (1973). See also McDonald (2003, Chapter 20).

The Bellman equation when the investor can invest in stocks, claims, and the bond is

$$\max_{C_t,\omega,y}\left[e^{-\rho t}U(C_t)+J_W E(dW)+\frac{1}{2}J_{WW}\begin{pmatrix}\omega' & y'\end{pmatrix}\mathbf{R}\begin{pmatrix}\omega\\ y\end{pmatrix}+J_t\right] \\ +\omega'\phi+y'\zeta+(1-\omega'\boldsymbol{\iota}-y'\boldsymbol{\iota})\psi \quad (30)$$

The Lagrange multipliers ϕ, ζ, and ψ are non-negativity constraints on ω, y and bonds.

Using equations (28) and (29), the first order condition for ω is

$$\omega' : J_W\left[\alpha(1-c_i)+\delta(1-d_i)-r\boldsymbol{\iota}(1-u_i)\right] \\ +J_{WW}W\left[\mathbf{V}\omega+\mathbf{V}\eta y\right]+\phi-\boldsymbol{\iota}\psi=0 \quad (31)$$

and that for y is

$$y' : J_W\left[\alpha^*(1-c_i)+\delta^*(1-d_i)-r\boldsymbol{\iota}(1-u_i)\right] \\ +J_{WW}W\left[\boldsymbol{\eta}\mathbf{V}\omega+\boldsymbol{\eta}\mathbf{V}\boldsymbol{\eta}y\right]+\zeta-\boldsymbol{\iota}\psi=0 \quad (32)$$

Because of the no-short-sale assumption, an investor must choose between investing in a claim and bonds or the underlying stock and bonds. To analyze this choice, premultiply equation (31) by $\boldsymbol{\eta}$, and subtract equation (32). This gives

$$(\boldsymbol{\eta}\alpha-\alpha^*)(1-c_i)+\boldsymbol{\eta}\delta(1-d_i)-\delta^*(1-d_i) \\ -(\boldsymbol{\eta}r\boldsymbol{\iota}-r\boldsymbol{\iota})(1-u_i)+\boldsymbol{\eta}\phi-\zeta+\boldsymbol{\iota}(\psi-\boldsymbol{\eta}\psi)=0 \quad (33)$$

Using equation (27), we have

$$\boldsymbol{\eta}\alpha-\alpha^*=\boldsymbol{\eta}\boldsymbol{\iota}r-r\boldsymbol{\iota}+\delta^*-\boldsymbol{\eta}\delta$$

Make this substitution in equation (33) and multiply to by -1:

$$(r\boldsymbol{\iota}-\boldsymbol{\eta}r\boldsymbol{\iota})(u_i-c_i)+\delta^*(c_i-d_i)+\boldsymbol{\eta}\delta(d_i-c_i) \\ -\boldsymbol{\eta}\phi+\zeta-\boldsymbol{\iota}(\psi-\boldsymbol{\eta}\psi)=0 \quad (34)$$

To interpret equation (34), recognize that when there when there is a no-short-sale constraint, the borrowing constraint is not binding, and there are two perfectly-correlated alternative investments, such as the stock and a note based on the stock, the investor will hold the instrument with the highest after-tax Sharpe ratio. The after-tax Sharpe ratio for the k^{th} stock is

$$\frac{\alpha_k(1-c_i)+\delta_k(1-d_i)-r(1-u_i)}{(1-c_i)v_k} \tag{35}$$

Using equation (27), the after-tax Sharpe ratio for the claim based on the k^{th} stock is

$$\frac{[r+\eta_k(\alpha_k+\delta_k-r)-\delta^*](1-c_i)+\delta^*(1-d_i)-r(1-u_i)}{\eta_k(1-c_i)v_k} \tag{36}$$

The Sharpe ratio for the claim exceeds that for the stock if

$$r(1-\eta_k)(u_i-c_i)+\eta_k\delta_k(d_i-c_i)+\delta^*(c_i-d_i)>0 \tag{37}$$

The left-hand-side of this inequality is the same as the square-bracketed term in equation (34).

The comparison of Sharpe ratios is a help in understanding the tax benefits of the structured note. To understand inequality (37), consider the case where $\delta^*=0$, so that the claim makes no payouts and suppose that $d_i \geq c_i$. The note is then preferred to the stock as long as the elasticity of the note with respect to the stock, η_k, is less than one. In this case, the claim effectively provides a less-levered way to hold the stock and still receive capital gains treatment on the position. If the investor were to delever by holding fewer shares and more bonds, interest income on the bonds would be taxed at the rate u_i. For the dealer, delevering entails creating a synthetic equivalent of the note that combines the stock with a bond position, but the note implicity converts some interest income to capital gains income, where it is taxed at a lower rate. This effect is measured by the first term. Conversely, the claim is disadvantaged if the elasticity exceeds one, since the implicit interest in the levered position is then deductible only at the capital gains rate. Dealer arbitrage will generally entail delevering since the arbitrage activity of converting interest income to capital gains is most valuable for the high tax bracket investors. The term multiplying δ_k measures the benefit of having dividend income taxed at the capital gains rate.[12]

[12] Note that in the 2003 Tax Act, the tax rate on dividend income was reduced to equal the capital gains rate. However, the law is written so that the reduced dividend tax rate is received only by an investor directly holding the stock. If a dealer were to hold the stock and pass the dividend through to an investor, the tax benefit would be lost *unless* the dividend were to receive capital gains treatment, as in a note where the final payment were based on the total return of the stock rather than just its price.

If the condition in equation (37) is satisfied for a given stock, then it must be the case that in equation (34) we have

$$\eta_k \phi_k - \zeta_k + \psi(1 - \eta_k) > 0$$

Suppose that the non-borrowing constraint is not binding, so that $\psi = 0$. Then $\phi_k > 0$ so the investor does not hold the stock and may hold the structured note. The investor will hold the structured note if the no-borrowing constraint, ψ, is zero. The investor would short-sell the stock against the claim if possible. If the non-borrowing constraint is binding and $\eta_k < 1$, then the magnitude of the stock short-sale constraint must be reduced. When the borrowing constraint is binding, it is even possible that both ϕ_k and ζ_k can be zero, i.e. the investor holds both the stock and the claim. The basic tension is that for tax reasons, investors would like dealers to create claims that combine a large holding of bonds with a small number of shares in order to maximize the tax advantage of converting interest income to capital gains income. However, if the claims are too delevered the investor will run against the borrowing constraint while trying to take sufficient risk by holding claims alone. The investor will then replace some of the claim with the stock in order to increase risk-bearing. In fact the ideal for the investor would be for the bank to create a risk-free instrument taxed like a stock, but I assume this is not feasible. Thus, a representative high-bracket investor would like the claim designed so that η_k is as small as possible, consistent with being taxed at the capital gains rate.

5.2 Equilibrium With Dealers

In this section we assume that dealers issue claims with different elasticities. We want to we see how this affects equilibrium and unanimity.

The implication of equation (37) is that if dealers create securities with $\eta < 1$, and if investor short-sale and non-borrowing constraints are not binding, then investors will replace their holdings of stocks and bonds with claims. It will not be surprising that different investors have demands for claims with different characteristics.

The first-order condition for y, equation (32), reduces to that for ω when the elasticty of the structure, η, equals one. Since the investor holds either the stock or the claim, I will use equation (32) with $\omega = 0$ in this analysis.

The first-order condition for holdings of the claims is

$$y_i W_i = \frac{-J_W}{J_{WW}(1 - c_i)} [\boldsymbol{\eta_i} \mathbf{V} \boldsymbol{\eta_i}]^{-1} [\alpha_i^* + \delta_i^* - r\boldsymbol{\iota}] \tag{38}$$

The subscript on $\boldsymbol{\eta}$ permits investors in different tax brackets to hold different claims.

Now substitute equation (27) for α^* in equation (38), and pre-multiply by $\boldsymbol{\eta}_i$. This gives

$$\boldsymbol{\eta}_i y_i W_i = A_i \mathbf{V}^{-1}\left[r(\boldsymbol{\eta}_i^{-1}\boldsymbol{\iota} - \boldsymbol{\iota}) - r\boldsymbol{\eta}_i^{-1}\frac{1-u_i}{1-c_i} + \alpha + \delta + \boldsymbol{\eta}_i^{-1}\delta_i^*\frac{c_i - d_i}{1-c_i}\right] \quad (39)$$

Aggregate across investors and substitute for $\mathbf{V}$ from equation (4) and for α from equation (2).

$$\sum_i \boldsymbol{\eta}_i y_i W_i = \sum_i A_i \mathbf{S}[\mathbf{K\Omega K}]^{-1}\mathbf{S}\Bigg[r(\boldsymbol{\eta}_i^{-1}\boldsymbol{\iota} - \boldsymbol{\iota}) - r\boldsymbol{\eta}_i^{-1}\frac{1-u_i}{1-c_i}$$
$$+ \mathbf{S}^{-1}\left[X - \mathbf{S}\delta - r(1-\tau)B\boldsymbol{\iota}\right] + \delta + \boldsymbol{\eta}_i^{-1}\delta_i^*\frac{c_i - d_i}{1-c_i}\Bigg] \quad (40)$$

Notice that $y_i W_i$ is the i^{th} investor's dollar holding of the m claims. Hence from equation (24),

$$\sum_i \boldsymbol{\eta}_i y_i W_i = \mathbf{S}$$

Equation (40) essentially says that when there are structured claims, the sum of *implicit* shares—which is the quantity of shares held by the dealer hedging the claim—must equal supply.

Rewriting equation (40) by dividing by A and by adding and subtracting $Sr\boldsymbol{\iota}A_u$, we obtain

$$X\boldsymbol{\iota} + rB\boldsymbol{\iota}\left[\frac{A_u}{A} - (1-\tau)\right] + \frac{1}{A}\mathbf{S}\sum_i A_i\boldsymbol{\eta}_i^{-1}\delta_i^*\frac{c_i - d_i}{1-c_i} +$$
$$\frac{1}{A}\sum_i r\mathbf{S}A_i\left(\boldsymbol{\eta}_i^{-1} - 1\right)\left(1 - \frac{1-u_i}{1-c_i}\right) = r\frac{A_u}{A}K\boldsymbol{\iota} + \frac{1}{A}\mathbf{K\Omega K}\boldsymbol{\iota} \quad (41)$$

or

$$\mathbf{S}\alpha + \mathbf{S}\delta + \frac{1}{A}\mathbf{S}\sum_i A_i\boldsymbol{\eta}_i^{-1}\delta_i^*\frac{c_i - d_i}{1-c_i} +$$
$$\frac{1}{A}\sum_i A_i r\mathbf{S}\left(\boldsymbol{\eta}_i^{-1} - 1\right)\left(1 - \frac{1-u_i}{1-c_i}\right) = r\frac{A_u}{A}\mathbf{S}\boldsymbol{\iota} + \frac{1}{A}\mathbf{K\Omega K}\boldsymbol{\iota} \quad (42)$$

Equation (41) has an easy interpretation in one special case. Suppose the elasticity for each claim is set equal to that of the firm, i.e., $\eta = S/K$.

Then $\eta^{-1} - 1 = K/S - S/S = B/S$. Thus, equation (41) reduces to

$$X\boldsymbol{\iota} + \tau r B\boldsymbol{\iota} + \delta_i^* K\left(\frac{A_d}{A} - 1\right) = r\frac{A_u}{A}K\boldsymbol{\iota} + \frac{1}{A}\mathbf{K\Omega K}\boldsymbol{\iota}$$

When the elasticity of the claim is set equal to that of the firm, there are three straightforward tax effects. First and most important, the effect related to debt is simply the interest deduction, $\tau r B$. The Miller condition vanishes because the dealer has laundered interest income, turning it into capital gains. Second, there is the tax disadvantage of paying dividends. Finally, there is a tax adjustment on the certainty equivalent return earned by the firm, rK. Equation (41) can be compared with equation (14) to see the effect of the dealer.

5.3 Optimal Elasticity

The analysis in the preceding section takes as given η, the elasticity of the claim. In practice, investors can either purchase a contingent claim with the desired elasticity, or construct one by trading in options along with the stock. In this section I examine the characteristics of the optimally-selected elasticity.

Let $\omega = 0$ and maximize equation (30) with respect to η. The first-order condition is

$$J_W\frac{\partial\alpha^*}{\partial\eta} + \frac{1}{2}J_{WW}\frac{\partial(dW^2)}{\partial\eta} \geq 0$$

This can be rewritten

$$J_W W(1-c_i)\left[(\alpha + \delta - r\boldsymbol{\iota}) + \frac{\partial\delta^*}{\partial\eta}\left(\frac{1-d_i}{1-c_i} - 1\right)\right] + J_{WW}W^2(1-c_i)^2\mathbf{V}\boldsymbol{\eta}y \geq 0 \quad (43)$$

Use equation (38) to substitute for y in equation (43). After simplifying, we obtain

$$J_W W(1-c_i)\boldsymbol{\eta}^{-1}\left[r\boldsymbol{\iota}\left(\frac{1-u_i}{1-c_i} - 1\right) + \left(\delta^* - \frac{\partial\delta^*}{\partial\eta}\right)\left(1 - \frac{1-d_i}{1-c_i}\right) - \frac{\zeta - \boldsymbol{\iota}\psi}{1-c_i}\right] \geq 0 \quad (44)$$

Without restrictions on the elasticity, this holds with equality. While it is not obvious why a claim need pay dividends, given their disadvantageous tax treatment, equation (44) shows that any induced change in the dividend yield associated with changing the elasticity of the claim will have an effect on optimal elasticity.

To understand equation (44), suppose that $\delta^* = 0$. If $u_i > c_i$, the tax term multiplying the interest rate is positive. This measures the incentive of the investor to reduce the elasticity of a position, increasing the fraction of interest income taxed as capital gains. By reducing elasticity, the investor is substituting away from risk-free bonds into the structured claim. Once the explicit holding of bonds reaches zero, the borrowing constraint becomes binding so that $\psi > 0$. At this point the investor stops reducing the elasticity of the position. When the investor can choose elasticity, therefore, the optimal *explicit* holding of bonds is zero.

Suppose that equation (44) holds with equality, solve for $\zeta - \boldsymbol{\iota}\psi$, and substitute into the first order condition for y, equation (32). This gives

$$y_i W_i = \frac{-J_W}{J_{WW}(1-c_i)} [\boldsymbol{\eta_i} \mathbf{V} \boldsymbol{\eta_i}]^{-1} \left[\alpha_i^* + \delta_i^* + \frac{\partial \delta^*}{\partial \eta} \left(\frac{1-d_i}{1-c_i} - 1 \right) - r\boldsymbol{\iota} \right] \quad (45)$$

When it is possible to select the dividend yield independently of the elasticity, $\partial \delta^* / \partial \eta = 0$, and taxes do not appear in the first-order condition except multiplying J_{WW}: Investors in higher tax brackets face less after-tax risk for a given portfolio. There is no tax tilting of portfolios.

5.4 Unanimity with Dealers

We re-examine shareholder unanimity in this setting. Unlike in Section 4.4, there are two different ways to evaluate the effect of a change in debt. First, we can suppose the investor holds a claim and we then vary debt, keeping fixed the elasticity of the claim with respect to the stock price. This corresponds to the idea that investors are holding some structured claims, but debt continues to be a marginal investment. Second, we can vary the elasticity of the claim as the firm changes debt. This second experiment captures the idea that claims permit the investor to pay taxes on interest at the capital gains rate, and claims are restructured as firms change capital structure. I conduct this second experiment in two ways. First I have the investor optimally choose elasticity, as in Section 5.3. Second, I assume that dealers adjust the terms of the claim when the firm changes its leverage. In both cases there is no marginal personal tax disadvantage to debt.

5.4.1 Fixed Claim Elasticity

The change in utility for investor i due a change in B_k is given by equation (16). Using the definition of α_k^* and α, the derivative of α_k with respect to B_k is

$$\frac{\partial \alpha_{i,k}^*}{\partial B_k} = \eta_{i,k}\frac{\partial \alpha_k}{\partial B_k} = \eta_{i,k}\frac{1}{S_k}[\alpha_k + \delta - r(1-\tau)] \tag{46}$$

We also have

$$\frac{\partial (dW)^2}{\partial B_k} = 2\frac{1}{S_k^2}\omega_k(1-c_i)^2 K_k \eta_{i,k} J_{WW} W^2 \Omega_k \mathbf{K} S^{-1} \times \left(-\frac{J_W}{(1-c_i)WJ_{WW}}\right)[\boldsymbol{\eta}_i \mathbf{V} \boldsymbol{\eta}_i]^{-1}\left(\alpha_i^* + \delta_i^*\frac{1-d_i}{1-c_i} - r\boldsymbol{\iota}\frac{1-u_i}{1-c_i} + \frac{\zeta - \boldsymbol{\iota}\psi}{1-c_i}\right) \tag{47}$$

We can eliminate α^* in equation (47) by multiplying through by $\boldsymbol{\eta}^{-1}$. Doing this and substituting equations (46) and (47) into equation (16), the total change in utility with a change in B_k is

$$\omega_k W J_W (1-c_i)\frac{\eta_{i,k}}{S_k^2}\Bigg\{S_k[\alpha_k + \delta - r(1-\tau)] - K_k\Omega_k \mathbf{K}[\mathbf{K\Omega K}]^{-1}\mathbf{S}\boldsymbol{\eta}_i^{-1}\left[\alpha_i^* + \delta_i^*\frac{1-d_i}{1-c_i} - r\boldsymbol{\iota}\frac{1-u_i}{1-c_i} + \frac{\zeta-\boldsymbol{\iota}\psi}{1-c_i}\right]\Bigg\} \tag{48}$$

Using equation (42) to substitute for α_k and α we obtain

$$\begin{aligned}
&\omega_k W J_W(1-c_i)\frac{\eta_{i,k}}{S_k^2}\Bigg\{rS_k\frac{A_u}{A} + \frac{1}{A}K_k\Omega_k\mathbf{K}\boldsymbol{\iota} - \frac{1}{A}S_k\sum_j A_j\eta_{j,k}^{-1}\delta_j^*\frac{c_j-d_j}{1-c_j} \\
&\quad + \frac{1}{A}rS_k\sum_j A_j(\eta_{j,k}^{-1}-1)\left(\frac{1-u_j}{1-c_j}-1\right) - r(1-\tau)S_k \\
&\quad - K_k\Omega_k\mathbf{K}[\mathbf{K\Omega K}]^{-1}\Bigg[r\mathbf{S}(\boldsymbol{\eta}^{-1}\boldsymbol{\iota}-\boldsymbol{\iota}) + r\mathbf{S}\frac{A_u}{A}\boldsymbol{\iota} + \frac{1}{A}\mathbf{K\Omega K}\boldsymbol{\iota} \\
&\quad + \mathbf{S}\left(\boldsymbol{\eta}^{-1}\delta_i^*\frac{c_i-d_i}{1-c_i} - \frac{1}{A}\sum_j A_j\boldsymbol{\eta}_j^{-1}\delta_j^*\frac{c_j-d_j}{1-c_j}\right) \\
&+ \frac{1}{A}r\mathbf{S}\sum_j A_j(\boldsymbol{\eta}_j^{-1}-1)\left(\frac{1-u_j}{1-c_j}-1\right) - S\boldsymbol{\eta}_i^{-1}r\boldsymbol{\iota}\frac{1-u_i}{1-c_i} + \mathbf{S}\eta_i^{-1}\frac{\zeta-\boldsymbol{\iota}\psi}{1-c_i}\Bigg]\Bigg\}
\end{aligned} \tag{49}$$

Once again I consider the special case where stock k is uncorrelated with other stocks. In this case the derivative is proportional to

$$r\tau S_k \; + \; S_k\eta_k^{-1}r\left(\frac{1-u_i}{1-c_i}-1\right) \; + \; S_k\eta_{i,k}^{-1}\delta_i^*\frac{d_i-c_i}{1-c_i} \; - \; S_k\eta^{-1}\frac{\zeta_i-\psi}{1-c_i} \quad (50)$$

In the special case in which $\eta_k = S_k/K_k$, we have

$$rS_k\left(\frac{1-u_i}{1-c_i}-(1-\tau)\right)+rB_k\left(\frac{1-u_i}{1-c_i}-1\right)+K_k\delta_i^*\frac{d_i-c_i}{1-c_i}-K_k\eta^{-1}\frac{\zeta_i-\psi}{1-c_i}$$

Because the elasticity of the claim with respect to the stock price is fixed, if the firm increases leverage, the investor will hold a smaller dollar amount of the claim and will replace the holding with taxable bonds. Thus, even though the claim launders the firm's interest expense, at the margin, the investor holds more bonds and bears the extra interest expense. Moreover, the investor also loses capital gains treatment on the marginal holding of the claim. This accounts for the second, negative, term. Finally by holding less of the claim the investor bears less of the tax-disadvantaged dividend.

5.4.2 Optimally Chosen Elasticity

When the investor chooses elasticity optimally (for example by trading options as well as the stock), condition (44) holds with equality. Using equation (44) to substitute for the multipliers in equation (50), and assuming that $\partial\delta^*/\partial\eta = 0$, we obtain

$$\begin{aligned}\frac{\partial \mathrm{E}(dJ)}{\partial B_k} &\simeq r\tau S_k + S_k\eta_k^{-1}r\left(\frac{1-u_i}{1-c_i}-1\right)+S_k\eta_{i,k}^{-1}\delta_i^*\frac{d_i-c_i}{1-c_i} \\ &\quad - S_k\eta_k^{-1}\left[-r\left(1-\frac{1-u_i}{1-c_i}\right)+\delta_k^*\left(1-\frac{1-d_i}{1-c_i}\right)\right] \\ &= r\tau S_k \end{aligned} \quad (51)$$

To see the intuition behind this result, recognize that the investor is holding derivative claims and is borrowing-constrained. When the firm changes its leverage, the investor in reponse optimally changes the elasticity of the claims and offsets this change, implicitly holding more debt and less equity. The newly issued debt is taxed at the same rate as the equity it replaced, and the net benefit is therefore the tax deduction at the level of the firm.

5.4.3 Claim Elasticity Varies with Leverage

Suppose the claim is constructed so that the elasticity is proportional to that of the firm, say $\eta = \mu S/K$. Then $\eta S^{-1} = \mu/K$ is a constant. This greatly simplifies the calculation of the change in utility. Note that the $E(dW)^2$ term in equation (16) contains the expression $\mathbf{S}^{-1}\boldsymbol{\eta}$, and hence is unchanged by leverage changes. Thus it is only necessary to evaluate the change in expected wealth, in particular the change in α^*. The calculation in this section will assume that the shareholder is not borrowing constrained and hence invests only in the structured claim and taxable bonds.

From equations (2) and (27), we have

$$\alpha_k^* = r + \eta_k \left[S_k^{-1}(X_k - S_k\delta_k - r(1-\tau)B_k) + \delta_k - r\right] - \delta_k^* \tag{52}$$

Using the facts that $\partial(\eta_k S_k^{-1})/\partial B_k = 0$ and $\partial\eta/\partial B_k = -\mu/K$, we have

$$\begin{aligned} \frac{\partial \alpha^*}{\partial B_k} &= -\eta_k S_k^{-1} r(1-\tau) - \frac{\partial \eta_k}{\partial B_k} r \\ &= -\frac{\mu}{K_k} r(1-\tau) + \frac{\mu}{K_k} r = \tau r \frac{\mu}{K} > 0 \end{aligned} \tag{53}$$

Thus, when the claim elasticity varies with leverage, shareholders unanimously support increases in leverage.

6 Structuring Issues

In this section I discuss some of the practical issues associated with structuring claims to eliminate personal holdings of debt. The model assumes the ability to trade continuously to maintain the appropriate elasticity, which is to say, the appropriate implicit debt and equity mix. There are reasons this continuous trading is likely to be impractical. There is the obvious consideration of transaction costs. In addition, even if an asset receives capital gains treatment, capital gains on positions held less than one year are taxed at the ordinary income tax rate, just like interest. Thus structured claims need to be held for at least a year in order to achieve tax benefits. If an investor has some claims that have been held more than a year and some less, it is possible to be taxed at the long-term rate by specifically selling those that have been held for more than a year.

At a point in time there are numerous ways to create a position with a given elasticity. Figures 1, 2, and 3 depict three positions with a price of \$30 and an elasticity of 0.55, with the stock price at \$30.

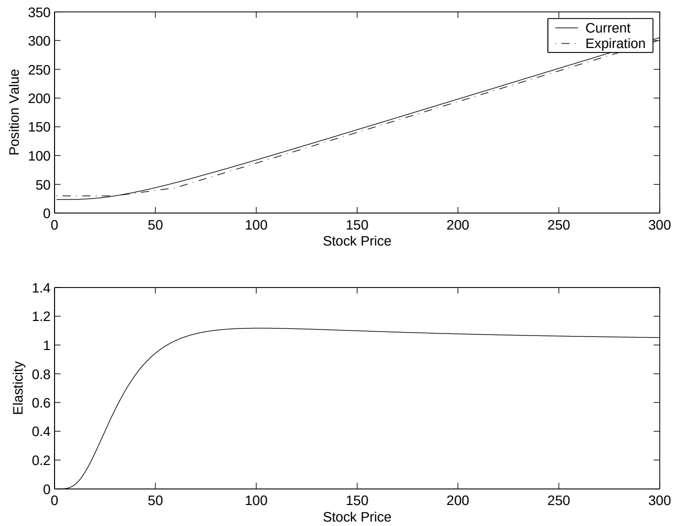

Figure 1: Payoffs and elasticity for a zero-coupon bond with promised payment of \$30, plus 0.468 purchased 30-strike calls, plus .602 purchased 60-strike calls, all with 4 years to maturity. The elasticty at \$30 is 0.55 and the price of the structure is \$30. Assumes $\sigma = 0.30$ and $\delta = 0$.

Figure 1 depicts a structured note. This is constructed as a bond, plus a fractional purchased call, plus a higher-strike fractional purchased call.[13] Since there is a minimum promised payment of \$30, this structure would presumably be taxed as contingent debt and therefore of interest only to tax-exempt investors. Note that as the stock price rises, implicit debt in the claim falls to zero and it provides an equity-like return. The need to trade to reestablish implicit debt is another reason the claim would be more attractive to tax-exempt investors.

Figure 2 illustrates a different claim with a price of \$30 and an elastic-

[13]The structure was constructed as follows. Let the quantities of the two options be a_1 and a_2. The value of the position is

$$V = De^{-rT} + a_1 C_1 + a_2 C_2$$

The elasticity of the position is

$$\eta = \frac{a_1 \Delta_1 + a_2 \Delta_2}{V}$$

Pick the desired elasticity, η, the two strike prices, $K_1 = \$30$ and $K_2 = \$60$, and pick the expiration date T and D. Given this information, only a_1 and a_2 are unknown. They can be found by solving two equations in two unknowns.

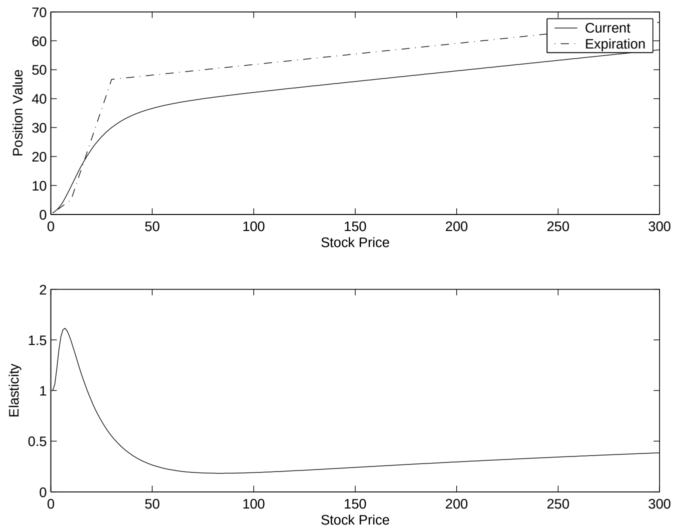

Figure 2: Payoffs and elasticity for 0.5 purchased shares of stock plus 1.584 written 10-strike calls and 2.01 written 30-strike calls, all with 4 years to maturity. The elasticty at \$30 is 0.55 and the price of the structure is \$30. Assumes $\sigma = 0.30$ and $\delta = 0$.

ity of 0.55 constructed using one-half share of non-dividend-paying stock, 1.584 purchased 10-strike options, and 2.01 written 30-strike options. This structure could possibly qualify as a "kinky prepaid forward", since it has no minimum promised payment despite having substantial implicit debt.

Both Figures 1 and 2 illustrate that the elasticity of the structures varies with the stock price. This change in elasticity can be reduced by adding additional options to the positions. Another alternative is a constant elasticity prepaid forward, paying S^a at expiration. Figure 3 illustrates the payoff of holding 5.38 units of this claim, for a cost of \$30 and elasticity of 0.55.

The varying elasticity of these claims should be seen in the context of equilibrium, which requires that investors in the aggregate—either directly or indirectly—hold the debt and equity issued by corporations. The question is whether at the margin, investors absorb these changes by holding debt and equity directly or by altering their holdings of claims.

To consider the issue of feasability, suppose that there are two classes of investors, taxable and tax-exempt, with equal amounts of wealth, and suppose there is a single firm with an equity-to-asset ratio of 0.55. It is possible for the tax-exempt investor to hold a structure like that in Figure 1 and the taxable investor to hold the structure in Figure 2. This would

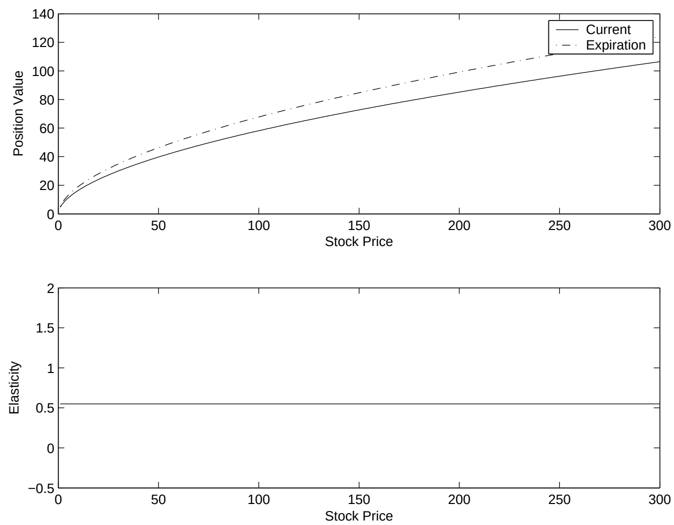

Figure 3: Payoffs and elasticity for 5.38 prepaid forwards paying $S^{0.55}$ after 4 years. The elasticty at all prices is 0.55. Assumes $\sigma = 0.30$ and $\delta = 0$.

be desirable for several reasons. First, given the contingent debt rules, the structure in Figure 1 would be best suited for a tax-exempt investor. Second, it might make sense that the two structures should add together to give the total value of the firm for every stock price. It is not always feasible to accomplish this however, which suggests that dynamic trading is a key to eliminating the bondholder tax. With dynamic trading, the two kinds of investors could hold structures that would become infeasible for both to continue to hold after large stock price changes—the payoff diagrams might not add up over the full range of possible stock prices to equal the payoff for the firm as a whole. This is not a problem as long as investors alter their position as the stock price changes. As mentioned above, however, dynamic trading can increase the fraction of the return taxed as short-term capital gains and hence like interest, which eliminates the reason for the structure in the first place. Thus, the design problem is one of trying to control the change in elasticity but recognizing that trading is inevitable.

In practice it may be that tax-minimizing structures make the most sense for wealthy taxable investors who can hold a portion of their portfolio in the form of structured claims. For example, a common strategy sold by banks to wealthy investors is a covered call. Figure 4 shows the price and elasticity for a covered call. The tax advantage of this position does not appear to have been generally recognized, but the analysis in this paper provides a tax

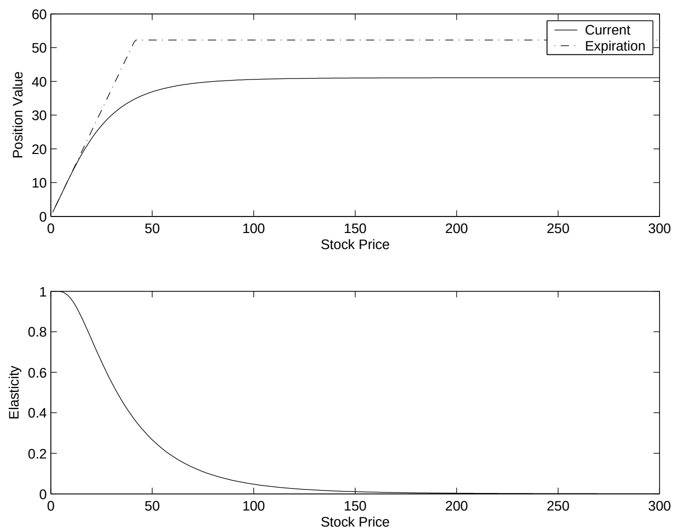

Figure 4: Payoffs and elasticity for 1.257 shares of stock and 1.257 written calls with a \$41.55 strike price and 4 years to expiration. The cost of the position is \$30 and the elasticity at a stock price of \$30 is 0.55. Assumes $\sigma = 0.30$ and $\delta = 0$.

rationale for the popularity of this and similar positions.

7 Conclusion

This paper explores the ramifications for corporate finance of the idea that investors can avoid the personal tax on interest by using derivatives. When this is possible, the Miller (1977) equilibrium reduces to the Modigliani-Miller equilibrium in which investors want firms to issue debt until the firm's marginal tax rate is zero.

In practice, there are a number of issues associated with ownership of structured claims as an alternative to direct ownership of stocks and bonds. Such claims generally do not have voting rights, they may have higher transaction costs and trade in less liquid markets, and there may be concern about a change in the tax law.

An additional consideration I have not discussed is credit risk. By owning structured claims instead of debt and equity, the investor replaces the firm's credit risk with that of the dealer. It is possible that credit derivatives could be structured to eliminate the credit risk of the dealer and replace it with the credit risk of the firm, but this would add another layer of cost to the transaction.

Finally, the recent growth (and proposed future growth) in tax deferred accounts may move firms away from a Miller equilibrium without the need for the derivative strategies discussed in this paper.

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