

Debt Valuation with Endogenous Default and Chapter 11 Reorganization

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ABSTRACT

We examine a continuous-time structural model of debt valuation with endogenous default and Chapter 11 bankruptcy. Focusing on exclusivity rules, we model the bargaining process in Chapter 11 as the debtor's ultimatum offers to the creditors and calibrate the model using an approach similar to that of Huang and Huang (2002). Credit spreads and expected Chapter 11 duration are shown to strongly depend on the outcome of a bargaining game between the debtor and the creditors. Calibrated credit spreads are up to twice as large as those produced by the model in Leland and Toft (1996) and explain the entire spread on speculative grade debt. We show, however, that credit risk and anticipation of Chapter 11 bankruptcy alone cannot explain the observed spreads on investment grade debt.

In addition, we obtain several new empirical implications of the model with regards to the expected time in bankruptcy as a function of different firm characteristics. The model predicts that firms with higher fraction of intangible assets, lower pre-bankruptcy volatility of asset value, and lower average maturity of debt in their capital structure are expected to spend less time in Chapter 11.

INTRODUCTION AND RELATED RESEARCH

*“In the last two years, more than 150 of the nation’s large public companies have trooped into bankruptcy court seeking protection from bondholders while they reorganize. The process, under Chapter 11 of the bankruptcy code, has long been praised as one that gives American capitalism an advantage over systems in other wealthy countries... But that hardly means that there is no room for improvement... The trouble is, Chapter 11 does not prevent various stakeholders and hired-hand advisers from pursuing their own interests at the expense of the company’s value – and lengthening the whole process.”*¹

A growing consensus among experts is that bankruptcy proceedings are too drawn-out and biased toward the debtor’s management, who usually stay in control of the failing business. This is aggravated by the fact that most traditional corporate governance mechanisms do not work in bankruptcy. Stocks and stock options, for example, are virtually worthless when a company is insolvent. As a result, firms may file for bankruptcy rather than continue operations because the debtor anticipates potential wealth transfers from bondholders rather than because continued operations are more economically efficient. In this paper we study the effect of this kind of strategic behavior by the debtor on value allocations, time in Chapter 11, and credit spreads by modeling the bargaining process in Chapter 11 as the ultimatum game of the debtor against the bondholders.

We develop a continuous-time valuation model of corporate debt with endogenous default and Chapter 11 bankruptcy.² In particular, we obtain closed-form solutions for (i) debt value and credit spreads, (ii) the allocations the claimants (the debtor and bondholders) receive as the outcome of the bargaining process in bankruptcy, and (iii) the endogenously determined timing of default and exit from bankruptcy period. We also calibrate our model to match the observed historical default rates and recovery experience by the bondholders. Recent work on

¹ “Chapter 11? Or time to close the books?” New York Times, December 15, 2002.

² Longstaff and Schwartz (1995) present a debt valuation model in stochastic short rate environment allowing for absolute priority rule violations. Saá-Requejo and Santa-Clara (1999) extend their analysis by modeling the liabilities (and, hence, the bankruptcy trigger) as an Ito diffusion process. However, both studies treat default as an exogenous rather than endogenous event resulting from optimal actions of both the debtor and the bondholders. Additionally, although write-downs in the event of default are specified in these studies, they are exogenous rather than resulting from modeling the expected outcome of a bargaining game between the debtor and the bondholders in the bankruptcy period.

this issue suggests that a number of existing credit risk models can predict only a *small* fraction of the observed yield spreads.³ We find using the calibration approach similar to that of Huang and Huang (2002) that incorporating bankruptcy into the model results in the higher explained fraction of the observed yield spreads. Under certain conditions, the model explains the entire speculative grade bond yield spreads that are observed in practice. Moreover, it produces credit spreads that are from twice to three times the credit spreads generated by the model of Leland and Toft (1996) for investment grade debt. Nevertheless, the model still explains only about 30-50% of the 10-year investment grade yield spreads. Furthermore, the calibration method produces higher credit spreads for firms that tend to be solvent at default and exhibit substantial violations of the absolute priority rules (APR).

Leland (1994) and Leland and Toft (1996) present valuation models of bonds with credit risk, which characterize a firm's optimal capital structure, with the bankruptcy trigger determined endogenously as the solution of the debtor's maximization problem. The models produce closed-form solutions for the value of risky debt, leverage, credit spreads, default rates, and write-downs. Goldstein, Ju, and Leland (1998) analyze optimal dynamic capital structure strategy by taking the claim to future EBIT as the underlying state variable and assuming that it is independent of capital structure. Leland (1998) encompasses both the elements of Modigliani and Miller (1958,1963) and Jensen and Meckling (1976) approaches to optimal capital structure. The model allows for the interaction of financing and investment decisions.⁴ These studies, however, assume that upon default the firm is liquidated with the bondholders obtaining the value of the firm and the debtor receiving nothing, which is at odds with the empirical evidence.⁵ Thus, the bankruptcy outcome is exogenously specified in these studies. In contrast, we obtain closed-form expressions for securities' values and credit spreads in a model allowing for strategic interactions between the debtor and the bondholders in the bankruptcy period. To achieve this, we combine

³ Elton et al. (2001) study different components of yield spreads using default data. Collin-Dufresne et al. (2001) find that economic variables that may determine default risk explain only a small fraction of yield spread movements. Finally, Huang and Huang (2002) conduct a comprehensive calibration study of a large class of structural models by fitting them to historical default experience data. They find that although the models explain most of the spread for speculative grade debt, they fail to account for more than about 20% of investment grade debt yield spreads.

⁴ Other similar models include Duffie and Lando (2001) who, modeling credit spreads, have allowed a stochastic process for the asset value to include imperfect observation, and Zhou (1996) who has introduced jumps. However, these studies focus on issues other than the impact of institutional features of the Chapter 11 process on strategic actions of the debtor and the bondholders.

the stationary debt structure of Leland and Toft (1996) with differential game theory applied to bargaining between the debtor and the bondholders in the bankruptcy period over allocations under a reorganization plan.

The focus of the strategic bargaining approach in the literature on pricing risky debt (Anderson and Sundaresan (1996), Mella-Barral (1999), Mella-Barral and Perraudin (1997), and Fan and Sundaresan (2000)) has been on strategic debt service.⁶ However, none of these studies consider the impact of Chapter 11 on the bankruptcy period allocations and, thus, on the debt values. They also obtain closed-form solutions only for the case of infinite maturity debt. Our approach is different from these studies in two important respects.

First, we model strategic bargaining in the bankruptcy period rather than before the firm enters the in-court bankruptcy period reorganization. The debtor chooses to default optimally, files for protection under Chapter 11, and remains in control of operations.⁷ The analysis of bargaining in Chapter 11 is complicated by the existence of a cram down provision, which is monitored by the judge.⁸ This provision affects the relative bargaining power of the debtor and bondholders. Our goal in this study is to learn how much more of the observed yield spread can be explained by the credit risk, if we incorporate Chapter 11 bargaining within a structural model. To this end, we assume that the debtor has all the bargaining power in the bankruptcy period.⁹ The possibility of cram down would only serve to reduce the credit spreads as it enhances the value of the outside option of bondholders. One consequence of these assumptions is that only the debtor may propose the reorganization plan. This is to a certain degree consistent with the low frequency of bondholders' plans documented by Weiss (1990) and Betker (1995)

⁵ For example, Franks and Torous (1989) report that equity holders get new equity in most Chapter 11 reorganizations. See also LoPucki and Whitford (1990) and Betker (1995).

⁶ See also Acharya et al. (1999).

⁷ Large financially distressed firms are rarely liquidated (see Garbade (2001)). Most often the debt is reorganized either out of court or in Chapter 11, which refers to in-court debt reorganization and is designed to protect the firms from bondholders' harassment and, as a result, premature liquidation. Management often stays in control of operations in Chapter 11.

⁸ The procedure of assessing whether a reorganization plan is fair and equitable to a dissenting class of claimants is known as a "cram down" hearing. A plan rejected by a class of claimants but confirmed by the court is said to be "crammed down" on the members of the dissenting class. A plan is "fair and equitable" to an impaired class of bondholders or the debtor if the plan respects the absolute priority rule with respect to that class.

⁹ More precisely, we assume that the exclusivity period is extended for the entire duration of bankruptcy. In other words, bondholders do not have cram down power. An accurate model of the effect of cram down would incorporate the asymmetry of the information about the underlying state variables among the debtor, the bondholders, and the court.

and recent evidence presented in Eraslan (2001), who estimated that the probability of the debtor being a proposer of a reorganization plan ranges from 61% to 77%.¹⁰ Therefore, unlike previous work, this study incorporates important institutional features such as the equity value maintenance in bankruptcy, Chapter 11 voting rules, and the consequences of the bargaining between the bondholders and the debtor for bankruptcy period allocations. Second, we obtain a closed-form solution for the valuation of finite maturity debt.

Bebchuk and Chang (1992) in a sequential bargaining model identify the expected outcome of the bargaining and examine the effects of the legal rules. However, they do not consider that both default and exit from bankruptcy are endogenous events. Recently, Morellec and Pascal (2002) modeled some Chapter 11 features. However, they do not model the bargaining power and time in Chapter 11 endogenously. Capital structure in their model contains only infinite maturity debt, which may lead to overstated credit spreads in their model for investment grade debt. Finally, they do not calibrate the model to match historical default experience.

The model also generates several empirical implications for the expected time firms spend in the bankruptcy period. We derive the average duration of bankruptcy as a function of firm-specific characteristics. The following trade-off is a key determinant of the expected time in bankruptcy. On one hand, given the default trigger value (i.e., the value at which default occurs), a longer stay in the bankruptcy period implies a higher payoff to the debtor at the reorganization boundary. On the other hand, this leads to a higher probability of liquidation and a zero payoff to the debtor. We find that the expected time in bankruptcy is a decreasing function of the liquidation cost and an increasing function of the debt maturity at origination and the volatility of the asset return in both the pre-bankruptcy and bankruptcy periods.

¹⁰ This probability is interpreted as the measure of bargaining power.

STRUCTURAL MODEL OF DEFAULTABLE DEBT WITH ENDOGENOUS DEFAULT AND CHAPTER 11 REORGANIZATION

2.1 The Model

In this section, we present a model that extends the work of Leland and Toft (1996) (hereafter, LT). The model allows us to examine the impact of both the strategic behavior of the debtor before default and bargaining between the debtor and bondholders on credit spreads.

We consider a firm with publicly traded debt. Agency problems between the manager of the firm and the shareholders are assumed away.¹¹ In this model managers' decisions reflect the solution to the debtor's optimization problem. There are two periods in the model: *the pre-bankruptcy period* and *the bankruptcy period* (see Fig. 1). In the pre-bankruptcy period, normal operations continue until financial health deteriorates to the point where it is no longer optimal for the debtor (the firm's shareholders) to meet scheduled coupon payments on debt obligations. In this paper, we assume that severe holdout problems render the successful private workout infeasible.¹²

We fix a filtered probability space $\{\Omega, \mathcal{F}, \mathcal{F}_{t>0}, \mathbf{P}\}$, where the filtration $\mathcal{F}_{t>0}$ is generated by the asset value process, $V \in \mathbb{R}_+$.¹³ At each level of the firm's asset value, V , the debtor has two alternatives: (i) she can keep paying coupons and dilute equity value, or (ii) she can stop debt payments and default. The latter event occurs at a stopping time $\tau_B = \inf\{t: V(t) = V_B\}$, where $V_B > 0$.

¹¹ For a description of associated problems and their impact on firm's investment decisions and debt value see Parrino et al. (2002)

¹² In practice, financially distressed firms can work out of distress through out-of-court debt renegotiation. This feature is considered in Anderson and Sundaresan (1996), Fan and Sundaresan (2000), Mella-Barral (1999), and Mella-Barral and Perraudin (1997). Achieving this kind of agreement among bondholders outside of the formal bankruptcy process depends on the type of debt being restructured, public or private (bank). We model only public debt held by a number of bondholders. The restructuring of public debt is governed by the Trust Indenture Act of 1939, which requires unanimous consent of every bondholder to alter bond indenture, coupon rate, principal or maturity. These strict voting rules make the restructuring difficult. A solution to these difficulties is often an exchange offer. However, the exchange often does not guarantee success because of holdout problems. In a typical exchange offer, bondholders are given an option to exchange their old claims for a package of securities, which are easier to service for a distressed firm. Since the participation is optional, there is always an incentive not to tender. In this case, if the post-exchange firm is considerably less distressed, the non-tendered claims will go up in value. Given that all bondholders have similar incentives and do not collude, the exchange often is likely to fail.

¹³ We assume that a complete measurable space $\{\Omega, \mathcal{F}\}$ is furnished with an increasing family of sub- σ -fields $\mathcal{F}_{t>0}$, $t \in \mathbb{R}_+$, i.e., $\mathcal{F}_s \subset \mathcal{F}_t$, for any $s < t$. This filtration is assumed to be right-continuous, i.e., $\mathcal{F}_t = \mathcal{F}_{t+} \equiv \bigcap_{\tau > t} \mathcal{F}_\tau$.

Once in default, the firm enters the bankruptcy period. Liquidation occurs only if the reorganization is unsuccessful. The bankruptcy period lasts until either of the following two events occur. First, the value of the firm's assets drops to an exogenously specified level $V=V_L$, at which a court appointed trustee liquidates the firm through conversion to Chapter 7. Second, the value goes up to an endogenously determined level $V=V_R$ at which a reorganization plan is proposed by the debtor.¹⁴ Liquidation occurs at time $\tau_L=\inf\{t:V(t)=V_L\}$ after default, if $\tau_L<\tau_R$. The plan is proposed at time $\tau_R=\inf\{t:V(t)=V_R\}$ after default provided $\tau_L>\tau_R$ (see Fig. 1). The bankruptcy trigger, V_B , and the reorganization triggers, V_R , are both determined endogenously to maximize equity value.¹⁵

2.1.1 Pre-Bankruptcy Period

Consider a firm that continuously sells a prespecified amount of new debt with maturity T to the public and simultaneously retires maturing debt. If the amount of new debt issue is not enough to finance the retirement of the maturing debt, then new equity is issued to cover the deficit. Otherwise, the excess is used to repurchase equity and/or to pay dividends. If default does not occur before an outstanding issue matures, the issue is retired at par. At any time t there is a continuum of issues outstanding with maturities ranging from 0 to T years with principal uniformly distributed over the interval $[t, t+T]$. The total amount (principal) of debt outstanding is P . Hence, debt is retired at a rate of P/T per year. The aggregate coupon payment by the firm on all issues currently outstanding is C per year. All debt issues have equal seniority. This is a modification of LT adapted to our two-period setting. The problem remains time-homogeneous. The benefit of this approach is that we do not have to solve for coupon that makes an issue to sell at par at origination. This reduces the dimensionality of numerical computations. Specifically, we have to solve only two equations for the triggers V_B and V_R , instead of solving three equations for both the triggers and the par coupon.

¹⁴ The decision to liquidate in Chapter 11 can be viewed as a random event with its stochastic trigger time equal to τ_L . Alternatively, liquidation can be modeled as an optimal decision by the court (see Baird and Morrison (1999))

¹⁵ LT maximize equity subject to its limited liability. In this model, however, equity value at the default boundary, $E(V_B)$, is positive due to the presence of bankruptcy and positive bargaining power of the debtor.

Note that these assumptions imply a stationary debt structure. As a result, none of the boundary conditions are time-dependent. The moving boundary problems (time-dependent) are reduced to free-boundary problems (time-independent). In particular, the bankruptcy boundary is time-independent.

It is assumed that there exists a risk-neutral probability measure, \mathbb{Q} , such that the evolution of the firm's asset value under this measure is expressed by the following diffusion process:

$$dV/V = (r - \delta 1_{t < \tau_B} - b 1_{t > \tau_B})dt + (\sigma 1_{t < \tau_B} + \omega 1_{t > \tau_B})dz, \quad (1)$$

where $r \in \mathbb{R}_+$ is the instantaneous risk-free rate of return, $\sigma, \omega \in \mathbb{R}_+$ are the instantaneous volatilities of the rate of return of the firm's assets in the pre-bankruptcy and bankruptcy period, respectively, $\delta \in \mathbb{R}_+$ is the fraction of the firm's assets paid out to debt and equity holders in the pre-bankruptcy period, and $b \in \mathbb{R}_+$ is the flow rate of bankruptcy costs in the bankruptcy period, z is a \mathbb{Q} -Brownian motion, and 1_A is the indicator function for event A .¹⁶

2.1.2 Bankruptcy Period

In this period equity is under the protection of the court.¹⁷ Once the debtor files for Chapter 11 protection, all payouts are frozen and the firm incurs a continuous stream of bankruptcy costs, bV , for a period of $\tau_L \wedge \tau_R$ years, where $x \wedge y \equiv \min(x, y)$.

The timing of the reorganization plan and allocations under the plan are endogenously determined by the debtor.¹⁸ The reorganization trigger is chosen by the debtor to maximize equity

¹⁶ In particular, δ is assumed to be independent of capital structure. Thus, investment policy is independent of financing decisions. For example, additional asset sales to meet debt service when V falls substantially are ruled out. In some previous studies (e.g., Kim et al. (1993)), when V falls to C/δ , the debtor is unable to meet debt service obligations, and, as a result, bankruptcy is declared (cash-flow default). In this paper, however, additional equity issues cover the deficit, if necessary.

¹⁷ In practice, under Chapter 11 provisions, all debt claims are stayed for at least 120 days during the so-called exclusivity period. The debtor will remain in control of the firm's assets for at least 6 months and often much longer. If the plan is accepted, new "softer" debt contracts are exchanged for the old "hard" contracts. During this period the debtor-in-possession is expected to formulate a reorganization plan, and no one else can propose a plan. The bankruptcy judge extends this exclusive period quite often. Additional delay is due to the fact that the plan acceptance period is 180 days within the bankruptcy filing. Only after that and only if acceptance has not been obtained can the bondholders propose a plan. However, unlike debtors, they must provide a costly appraisal of the firm's asset values to the court. These costs become virtually prohibitive for credit plans. Weiss (1990) found that there was only one case of credit plan among 37 firms examined.

value. We assume that the debtor (managers on behalf of all shareholders) is in control of the agenda. Betker (1995) mentions that “in practice, managers control this option [to delay a reorganization]...” (p. 161).

In this paper, the bargaining process between the debtor and the bondholders is modeled as follows. After default is declared, the court stays the bondholders’ claims. This ‘automatic stay’ continues until either the firm’s asset value drops to $V_L = \theta V_B$, where $0 < \theta < 1$ is an exogenously specified parameter, or managers propose a reorganization plan as soon as the asset value goes up to $V = V_R$, whichever comes first. If the firm’s asset value drops to V_L , then the firm is automatically liquidated. The firm is sold under the court supervision with the debtor getting nothing and the bondholders receiving the value of the firm net of liquidation costs, i.e., $(1 - \alpha)\theta V_B$, where $0 < \alpha < 1$ is an exogenously given percentage liquidation cost. The bondholders and the management play a dynamic ultimatum game in the bankruptcy period. As shown later, for parameter values for which the solution to the equity maximization problem exists, there exists a firm asset value, $V = V_R > V_B$, at which it is optimal for the debtor to propose the plan. Moreover, the plan is such that it will not be optimal for the bondholders to reject it.

The plan is confirmed in a court hearing, a costly process. At this point the firm is assumed to incur a one-time expense, K , which may include all or some of the following: all legal (e.g., lawyer fees associated with plan confirmation), accounting, brokerage, and investment banking fees. It is assumed here that bondholders are paid either in cash and/or equity through a debt-equity swap.¹⁹ After that the firm will operate as an all-equity entity.

¹⁸ In theory, a debtor’s reorganization plan must specify how each class of bondholders is to be treated, the cash and securities each class will receive, when they will receive them, and whether a particular class is impaired or not. The bankruptcy judge is critical for the decision process in that she confirms all the values in the plan. Use and precision of these values are often quite limited. Consequently, in practice, most plans do not exactly provide an estimate of what each class would receive in liquidation, but only state that the plan provides more. Further, the court accepts management valuations unless bondholders can furnish different values, a costly process. The court will accept the management opinion of whether a bondholder is impaired or not. It is quite costly for the bondholder to contest that (Weiss (1990)).

¹⁹ In this model, matters are deliberately simplified to make the model tractable. In reality, bondholders may get a variety of securities. According to Weiss (1990) and Gilson et al. (1990) bondholders can receive different combinations of cash, notes (both zero-coupon and coupon bearing), shares, warrants, and convertible securities.

2.1.3 Default Specification

In this paper, we endogenize both the bankruptcy boundary (value of the state variable at which the default is triggered) and the timing of exit from the bankruptcy period. Since the default event is completely controlled by the debtor, it will be chosen optimally so that it maximizes the initial value of the debtor's claims.²⁰ If the firm is illiquid (cash flows generated by the firm are insufficient to meet the debt service), default will not necessarily occur. The debtor may delay it by selling equity and covering the shortage of funds with proceeds from an equity issue. Eventually, the debtor will choose to default at a point where it is not optimal to raise new equity capital to meet net debt service requirements. This approach endogenizes the default boundary and has been used in Leland and Toft (1996), Mella-Barral and Perraudin (1997), Goldstein, Ju, and Leland (1998), and Leland (1998).

Different default-triggering mechanisms are considered in the literature. Some are related to certain covenants in bond indentures. A “positive net worth” covenant triggers default if the asset value falls to face value of debt, $V_B = P$. This approach has been implemented in Black and Scholes (1973), Merton (1974), Ingersoll (1977 a,b), Merton (1977), Smith and Warner (1979), and many others. Bankruptcy may be triggered by illiquidity, i.e., when net earnings after interest fall below zero.²¹ This approach has been used in Kim et al. (1993), and Anderson and Sundaresan (1996). However, these approaches ignore the fact that the debtor's incentives to optimize its own value can significantly impact default timing decision.

2.2 Pre-Bankruptcy Values

The total value of the firm consists of the value of the assets, V , tax shields, $TS(V)$, and bankruptcy costs, $BC(V)$. Let v denote the value of the levered firm. Then

$$v = V + TS - BC . \quad (2)$$

The value of equity is given by:

²⁰ Since we focus on the exclusivity rules of Chapter 11, we assume that the debtor also controls the exit from Chapter 11. As shown in Appendix A, this is not a binding assumption as long as there is no possibility of bondholders' cram down.

²¹ To preserve simplicity, loss carry-backs or carry-forwards are not modeled here.

$$E(V) = V + TS - BC - D. \quad (3)$$

2.2.1 Debt Value

Consider a bond that matures T years from now, paying continuous coupon C/T per year and principal P/T at maturity.²² At maturity the issue is retired at par. Let $d(V, t)$ denote the value of this bond when the value of the state variable is V , and the time remaining to maturity of the bond is t . Then, $d_B \equiv d(V_B)$ is the value of the bond at default. We assume that all debt issues have the same priority. Thus, at default the total value of debt, D_B , is equal to Td_B . The default trigger, V_B , is an unknown constant (not a function of time). It will be determined later as a solution of a sequential optimal stopping problem maximizing shareholders' value.

Following standard methods (see, e.g., Merton (1974)), the value of this debt issue of maturity t at $V_{s=0}=V$ can be shown to be the solution of the following partial differential equation:²³

$$.5\sigma^2 V^2 \partial_{VV} d + (r - \delta)V \partial_V d + \partial_s d - rd + C/T = 0 \quad (4)$$

subject to the following boundary and initial conditions:

$$\begin{aligned} d(\infty, s) &< \infty, \text{ for all } s > 0 \\ d(V_B, s) &= d_B, \text{ for all } s > 0 \\ d(V, T) &= P/T. \end{aligned} \quad (5)$$

The solution to (4)-(5) can be obtained using standard methods (see, e.g., Oksendal (1998)):

$$d(V, t) = E^V \left[\int_0^t \frac{C}{T} 1_{s < \tau_B} e^{-rs} ds + d_B 1_{t > \tau_B} e^{-r\tau_B} + \frac{P}{T} e^{-rt} 1_{t \leq \tau_B} \right], \quad (6)$$

where τ_B is the first passage time of the process $V(s)$ to V_B ; $E(*)$ is the expectation operator with respect to the process (1) written with respect to probability measure \mathbb{Q} . Equation (6) admits

²² The following discussion is an adaptation of LT to the present two-period setting. The uniform distribution of debt across maturities $[0, T]$ implies that an individual debt issue can be considered as a bond with principal P/T and coupon C/T .

²³ Due to the stationary debt structure in the model, at any two times, $s \neq s'$, the values of an individual debt issue, $d(s)$ and $d(s')$, are the same as long as the values of the state variable V are the same at these times, i.e., $V(s) = V(s')$. Thus, without loss of generality we can assume that initial moment coincides with $s=0$. In other words, the value of the state variable, V , is a sufficient statistic with respect to determining all other values.

simple interpretation. The first term is the expected present value of the coupon stream corrected for the probability of the default occurring before maturity, t . The second term is the expected present value of the bondholder's allocation at default. The last term is the expected present value of the principal adjusted for the probability that the issue matures before the default.

To find the total value of debt outstanding, it is convenient to view total debt as a combination of a continuum of individual issues. The total value of all debt issues outstanding at date 0 is given by:

$$D(V) = \int_0^T d(V, t) dt. \quad (7)$$

The following result (which is a modification of LT into the current two-period setting) presents closed-form solutions for (i) the value of debt issued at date t and (ii) the total value of debt outstanding. It provides a quantitative link between the values in the pre-bankruptcy period and those in the bankruptcy period.

Lemma. Let $X \equiv \ln(V/V_B)$. Then the value of each individual debt issue with original maturity of T years and t years left to maturity is given by:

$$d(X, t) = \frac{C}{rT} (1 - e^{-rt}) + \frac{P}{T} e^{-rt} + E1_{t > \tau_B} \left[\frac{C}{rT} - \frac{P}{T} \right] e^{-rt} + E1_{t > \tau_B} e^{-r\tau_B} \left[\frac{D_B}{T} - \frac{C}{rT} \right]. \quad (8)$$

The value of all debt outstanding is

$$D(X) = \frac{C}{r} + \left[P - \frac{C}{r} \right] \left[\frac{1 - e^{-rT}}{rT} - I_1(X) \right] + \left[D_B - \frac{C}{r} \right] I_2(X), \quad (9)$$

where

$$I_1(X) = \frac{1}{rT} \left[E1_{\tau_B < T} e^{-r\tau_B} - e^{-rT} E1_{\tau_B < T} \right],$$

$$I_2(X) = \frac{1}{\sigma v^- \sqrt{T}} \left[e^{\gamma_2^- X} N(\lambda_1) \lambda_1 - e^{\gamma_1^- X} N(\lambda_2) \lambda_2 \right],$$

$\mu^-, v^-, \gamma_1^-, \gamma_2^-, \lambda_1$, and λ_2 are defined in Table 1 (also see Appendix A), and $N(\cdot)$ is the standard normal cumulative distribution function.

The terms in equation (8) containing C represent the expected present value of coupon stream. Essentially, this is the sum of two continuous-time annuities - one of length T , the other

of length τ_B . The values of these annuities are corrected for the probabilities of the events $\tau_B > T$ and $\tau_B < T$, respectively. The remaining terms represent the expected present values of the principal and payment in default. Equation (9) represents the aggregate value of all individual debt issues outstanding.

2.2.2 Values of Tax Shields and Bankruptcy Costs

As in Leland (1994), Leland and Toft (1996), and Leland (1998), the firm is assumed to earn tax refunds in the form of debt tax shields. These tax benefits accrue at a rate τC per year until the firm is in default, where τ is the corporate tax rate. The value of the tax shields can be written as²⁴

$$TS(V) = E^V \left[\int_0^{\tau_B} \tau C e^{-rs} ds \right] = \frac{\tau C}{r} E \left[1 - e^{-r\tau_B} \right] = \frac{\tau C}{r} \left[1 - \left(\frac{V}{V_B} \right)^{\gamma_2} \right], \quad (10)$$

where γ_2 is defined in Table 1.

Not surprisingly, at default the value of tax shields vanishes. Beyond the bankruptcy point, $V = V_B$, it is identically zero. Note that this implies that $TS(V)$ is not a smooth function of V at $V = V_B$.

Let's denote the value of bankruptcy costs in pre-bankruptcy and bankruptcy periods as BC^- and BC^+ , respectively. As the flow of bankruptcy costs in the pre-bankruptcy period is zero, and the problem is stationary, BC^- obeys the following ODE:

$$.5\sigma^2 V^2 \partial_{VV} BC^- + (r - \delta) V \partial_V BC^- - r BC^- = 0, \quad (11)$$

with the following boundary conditions:

$$\lim_{V \rightarrow \infty} BC^-(V) = 0; \quad BC^-(V_B) = BC^+(V_B).$$

The first boundary condition says that as the firm becomes more and more successful, the value of bankruptcy costs is reduced and must vanish in the limit for large V . The second boundary condition is the continuity of BC across the free boundary $V = V_B$. Just like $TS(V)$,

²⁴ It is possible that EBIT falls below coupon value at some time before bankruptcy. In this case, portion of tax benefits is lost. I ignore this possibility, as it does not affect the results qualitatively (see LT or Leland (1998) for discussion).

$BC(V)$ is not a smooth function of V at $V=V_B$. The solution to (11) subject to the boundary conditions is given by:

$$BC^-(V) = BC^+(V_B) \left[\frac{V}{V_B} \right]^{\gamma_2}. \quad (12)$$

2.3 Bankruptcy Period Values

The value of the levered firm in this period consists of the value of the assets net of the expected present value of bankruptcy costs:

$$v(V) = V - BC^+(V). \quad (13)$$

The value of equity is then:

$$E(V) = v - D(V). \quad (14)$$

Since all payouts are frozen during the bankruptcy period, the dynamics of the asset value are different from that in the pre-bankruptcy period. In this case, the claimants to a portion of the firm's cash flows are lawyers and/or trustees appointed to supervise the bankruptcy process. Neither the debtor nor the bondholders have any cash distributions in this period. Given this, we write the risk-neutral asset dynamics in the bankruptcy period as follows:

$$\frac{dV}{V} = (r - b)dt + \omega dz, \quad (15)$$

where b is the rate at which bankruptcy costs are paid out. Formula (15) is a restriction of equation (1) on the bankruptcy period, i.e., when $\tau_B < t$.

2.3.1 Bankruptcy costs

The total bankruptcy costs consist of a flow component and a lump sum component. The flow component of the bankruptcy costs, accruing at a rate bV per year, is continuous and is taken into account in the drift coefficient in (15).²⁵ The lump sum component depends on two mutually

²⁵ Continuous flow of bankruptcy costs can be viewed as a payout stream to the holders of hypothetical claims against unlevered asset value.

exclusive events. If the value of the firm's assets, V , drops to θV_B , then the amount $\alpha \theta V_B$ is lost in liquidation. However, if the value increases substantially and the plan is eventually proposed, then a formal court hearing is necessary to examine and confirm the plan. This is a costly procedure. It involves a one-time cost to cover the expenses. There may also be accounting, brokerage, and investment banking fees. The total amount of these costs is assumed to be K .

2.3.2 Debt Value

Debt value as well as credit spreads and duration of Chapter 11 bankruptcy will critically depend on the way the bankruptcy period bargaining is modeled. In this paper, the focus is on the consequences of the exclusivity the debtor receives under Chapter 11. In a typical debt contract, the bondholder receives interest and principal as long as the firm is solvent. In default, the bondholders obtain the claim against the debtor's assets. The interest in equilibrium is determined so as to compensate the bondholders for expected losses in case of default.

In the absence of a bankruptcy system, each bondholder would exercise her own contractual right individually. This would result in a piecemeal liquidation of the debtor's assets and loss of any going concern value (synergies, growth opportunities, monopoly power, specific human capital, etc). The optimal corporate reorganization system must preserve the going concern value, if any, and distribute it to the bondholders in the form of cash and/or securities.²⁶

It is tempting to say that giving all the value to the debtor can maximize the going concern value. However, the bondholders will charge high interest rates anticipating zero payoff in case of default. This results in inefficient allocation. Thus, the bankruptcy law balances two objectives: preserving the going concern value and respecting pre-bankruptcy contracts. The reconciliation of these two conflicting objectives can be achieved by reorganizing the firm with bondholders with no post-bankruptcy interest receiving at least as much as they would in liquidation. The objective is achieved by information pooling through strategic voting.

In this model, the game is played in continuous time. Both parties, the bondholders and the debtor, have perfect information with respect to the dynamics of the state variable, V , as well as each other's valuations. The underlying asset value (and, thus, all contingent claims written on

it) is assumed to be observable to both parties, the bondholders and the debtor, but imperfectly observable by a third party, the court, and thus not verifiable. Therefore, the payoff of contracts cannot be conditioned and enforced based on observed values. The resolution is achieved only upon a mutual agreement of the debtor and the bondholders. The assumption of imperfect observation by the court, however, requires further elaboration.

The bankruptcy judge (unlike ordinary judges) can appoint an examiner to evaluate the firm (Baird and Morrison (1999)). There are, however, a number of reasons to believe that the firm specific information collected in this fashion is not sufficient enough to entrust the bankruptcy judge with an unconditional power to make reorganization and liquidation decisions. Bankruptcy judges are not subject to market discipline in that there are no competitive mechanisms punishing bankruptcy judges for bad decisions. They are subject to reappointment every 14 years by other federal judges who are not well suited to assess the bankruptcy judge's performance. At the same time, only a small part of their activity is spent dealing with bankruptcy cases. Thus, it is reasonable to expect that due diligence in making bankruptcy decisions may not be exercised appropriately.²⁷ In addition, there can be a cognitive bias that stems from the fact that bankruptcy judges are lawyers by training. As Baird and Morrison (1999) put it, most bankruptcy judges have no training even in the fundamentals of corporate finance. Further, even rules of judicial conduct reduce the bankruptcy judge's ability to obtain firm-specific information informally. According to Section 341 of the Bankruptcy Code, bankruptcy judges cannot attend the meetings where management of the firm must disclose information to the bondholders (Baird and Morrison (1999)).

In light of these potentially serious limitations of the third party arbitration system in bankruptcy, in practice it is structured as a bargaining process between the debtor and the bondholders. Thus, the final outcome is conditioned on the information revealed in the bargaining process.

The debtor plays an ultimatum game with bondholders. At any time, t , the debtor can make an offer to the bondholders or refuse to make an offer.²⁸ If an offer is made, the

²⁶ This redistribution may and often does cause the problem of pre-bankruptcy incentives. Equity may fail to maximize the firm value in anticipation of bankruptcy by over-investing in risky projects.

²⁷ Bankruptcy judges will typically know nothing about the firm at the beginning of the case. Even if they do, they may be forced to abandon the case on the grounds of conflict of interest.

²⁸ It is assumed that hypothetical trustee represents all bondholders.

bondholders may either accept or reject it. We restrict the role of the bankruptcy judge to plan confirmation decisions given the outcome of the bargaining. If the bondholders object, then the judge rejects the plan. In this case, the game continues to a later period, $t+dt$. If the bondholders accept the offer, the plan is confirmed, and the firm is reorganized with both parties receiving their equilibrium allocations as described in Theorem 1 below. Reorganization results in the firm's exit from bankruptcy and operation as an all-equity entity. Further, the game is ended by the court order automatically as soon as the asset value drops from V_B to θV_B .²⁹ At this point the firm is liquidated in Chapter 11 under court supervision with absolute priority rules being enforced.³⁰

To put formal structure on the game, we assume that each player determines her actions by the Markovian strategies $u=\phi(V)$.³¹ The evolution of the state variable is governed by stochastic differential equation (15) subject to initial condition $V_{t=0}=V_B$.³² A set of the bondholders' permissible strategies is represented by a binary decision variable $u_1(V): \mathbb{R}_+ \rightarrow \{0,1\}$, corresponding to the decision whether to reject or accept the debtor's offer, respectively. The debtor's decision variable is a continuous function of V , $u_2(V)=D^+(V): \mathbb{R}_+ \rightarrow \mathbb{R}_+$, representing the optimal amount to be offered by the debtor to the bondholders. As the strategies are Markovian, the optimization problem for each player is reduced to an optimal control problem (see Dockner et al. (2000)). The bondholders maximize the value functional of debt given the debtor's strategy, and the debtor maximizes her value functional given the bondholder's strategy.³³

²⁹ This assumption is reasonable. It reflects the fact that the bondholders may file a lawsuit claiming that the debtor failed to use the assets properly for reorganization purposes. The judge may rule in favor of liquidation after she receives a signal of a substantial drop in asset value and, as a result, low probability of recovery.

³⁰ As mentioned in Baird and Morrison (1999), there is no specific Bankruptcy Code provision for liquidation. Instead, what we call Chapter 11 liquidation can take several different forms, which are essentially tantamount to shutting the firm down. These forms include conversion of Chapter 11 to Chapter 7, granting a secured bondholder's motion to seize its collateral (11 U.S.C. §362(d)), or restricting the debtor's ability to obtain debtor-in-possession financing (11 U.S.C. §364).

³¹ There is no time dependence here, since the problem is time-homogeneous.

³² In the most general form of a differential game, the drift and volatility are functions of time, state variable, and controls (players' actions). In current formulation, the evolution of the state variable is time-homogeneous. Also, the process is purely exogenous, i.e., actions of the players do not affect the state variable dynamics. Thus, the drift and the volatility are not functions of players' actions.

³³ Stochastic differential equation (15) may, in general, not have a unique solution. It may have multiple state trajectories, and, thus, lead to possible non-uniqueness in the functionals for a single set of strategies $\{u_1, u_2\}$. However, under certain conditions, the solution is unique. These conditions can be found in Oksendal (1998). The conditions trivially hold for the special case considered here.

Optimal control problems are a special class of dynamic games with one player and one optimization criterion. Here, we adopt a concept of non-cooperative Markovian Nash equilibrium, in which each player is faced with a single criterion optimization (optimal control) with the strategies of the remaining players fixed at their equilibrium levels.

Definition. A pair of functions $\{\phi_1, \phi_2\}$ is a non-cooperative Markovian Nash equilibrium solution for a two-player non-zero sum dynamic game if for each $i \in \{1, 2\}$ an optimal control path $u^i(\cdot)$ of the following Nash condition

$$W^i(V) = \sup_{u^i \in U_{\phi^{-i}}^i(V)} J^i(V, u^i, u^{-i})$$

exists and is given by the Markovian strategy $u^i = \phi_i(V)$. Here, $W^i(V)$ is the optimal value of player i 's objective functional, $J^i(\cdot)$ is player i 's objective functional expressed as a function of the players' strategies and the value of the firm's assets, and $U_{\phi^{-i}}^i(V)$ is the space of admissible strategies by player i given the opponent's equilibrium strategies.

In the case of the non-cooperative Nash equilibrium considered here, the term “equilibrium” means a consistent prediction of the outcome of the game. When player i rationally expects the rivals to play their Nash equilibrium strategies, she can do no better than choose her own Nash equilibrium strategy. As all players are assumed to be rational and aware that all the rivals are rational, no player would do anything else but choose her Nash equilibrium strategy. Thus, each player maximized her objective given the equilibrium strategies of her opponents. The following proposition provides a formal solution to the bankruptcy period game.

Theorem 1. In the bankruptcy period game, a non-cooperative Markovian Nash equilibrium defined above can be characterized as follows. The debtor's equilibrium strategy, if she makes an offer at V , is to offer

$$\phi_2 = D(V) = (1 - \alpha)\theta V_B \left(\frac{V}{\theta V_B} \right)^{\gamma_2^+}, \quad (16)$$

where α and θ are as defined before, and

$$\gamma_2^+ = -\frac{r-b-.5\omega^2}{\omega^2} - \sqrt{\frac{r-b-.5\omega^2}{\omega^2} + \frac{2r}{\omega^2}}.$$

The bondholders accept the offer, $\phi_1=1$. The Nash outcome of the game is given by:

$$W^1(V)=D(V)=\phi_2(V); \quad W^2(V)=V-BC^+(V)-\phi_2(V). \quad (17)$$

In equilibrium the optimal strategy by the debtor is to offer slightly more than the lowest possible outcome to the bondholder under the reorganization plan. In Chapter 11 a plan will not be confirmed if any bondholder who objects to the plan receives less than it would under Chapter 7 (liquidation floor). The equilibrium allocation to the bondholders, according to Theorem 1, is always lower than what they would receive upon immediate liquidation of the firm. Thus, one might assume that the court would strike down a plan offering an amount just above ϕ_2 . The problem with this argument is, though, that the Bankruptcy Code requires the court to enforce the liquidation floor only if a bondholder objects to the plan (11 U.S.C. §1129(a)(7)(A)(i)). But if the debtor keeps the agenda control, as is the case in the exclusivity period, the bondholders would not object. Otherwise, they may never be offered more than the debtor's initial offer ϕ_2 . The above argument is based on the assumption that the bankruptcy period is identical to the exclusivity period, which is routinely extended by the court.³⁴ Another way for the equity holders to avoid the enforcement of the liquidation floor and have their plan confirmed is misrepresentation of the liquidation value. "Managers of a bankrupt firm face no legal or financial penalties for misrepresenting the firm's liquidation value in the disclosure statement: Section 1145(a) of the Bankruptcy Code exempts firms in Chapter 11 from ordinary registration and disclosure requirements under federal and state securities laws" (Gilson (1997), p. 181).

2.3.3 Equity Value

Let E^+ denote equity value in the bankruptcy period and $V_L=\theta V_B$ be the value of the assets at liquidation. Also, let $X_L = \ln(V/V_L)$ and $\tau_L = \inf\{t>0: X_L=0\}$. Thus, X_L is a transformed state variable that becomes equal to zero at liquidation, and τ_L is the stopping time of this event.

Let V_R denote the value of the assets at which the reorganization plan is proposed. Then, the value of equity is given in

Theorem 2. Let $\chi = V_R/V_L$, $X_R = \ln \chi$, and $\tau_R = \inf\{t > 0 : \ln(V_R/V) = 0\}$. Then equity value

$$E^+(V) = (V_R - K - (1 - \alpha)V_L \chi^{\gamma_2^+}) \xi_1(V), \quad (18)$$

where

$$\xi_1 = \frac{e^{\gamma_1^+ X_L} - e^{\gamma_2^+ X_L}}{e^{\gamma_1^+ X_R} - e^{\gamma_2^+ X_R}}. \quad (19)$$

In our model the debtor's claim is a sequential exercise option. In the pre-bankruptcy period it is analogous to the claim the debtor holds in the LT model, but in the bankruptcy period it is a perpetual down-and-out American call option with exercise value endogenously depending on all of the model parameters.

2.3.4 Determination of Default and Reorganization Triggers

Until early in the 1990s, the stopping boundaries in debt pricing problems were mostly exogenously specified. In Merton (1974) bankruptcy occurs only at maturity (when $V < P$).³⁵ Kim et al. (1993) assumed the exogenous (cash flow) boundary: default occurs when the cash flow available for payout falls below the required coupon payment for the first time. Longstaff and Schwartz (1995) and Saa-Requejo and Santa-Clara (1999) have studied bond pricing assuming that default occurs as soon as the asset value drops below an exogenously specified value.³⁶ Thus, these studies have failed to correctly model the bankruptcy boundary as the result of the optimal decision by debtors to pass control to bondholders.

Similarly to Leland (1994), Leland and Toft (1996), and Leland (1998), we endogenize the bankruptcy boundary. As shown by Leland and Toft (1996), whenever the cash flow available

³⁴ Managers always try to use the exclusivity to extract concessions from the bondholders. They quite often are able to get the period extended by the court. For example, in a study by Betker (1995) the exclusivity period was extended for the entire length of the bankruptcy in 80% of the cases considered (60 out of 75 firms).

³⁵ Merton analyzes zero-coupon bonds, however. In this case, there is no endogenous reason to default before maturity.

³⁶ In Saa-Requejo and Santa-Clara (1999) this exogenous boundary follows a certain exogenous stochastic process.

for payout is insufficient to meet debt service, the shortage is financed by new equity issuance. The debtor is willing to tolerate this dilution of her claims as long as the risk-neutral expected equity appreciation exceeds the cash flows that must be contributed to keep the firm out of bankruptcy.

As shown later, the decision to default by the debtor is not independent of its tactics in bankruptcy. Thus, the bankruptcy boundary, V_B , will depend on the timing (measured by V_R) of the reorganization plan proposal as well as the amount to be allocated to the bondholders under the plan. To determine the timing of both the optimal default and the reorganization plan proposal, a joint optimal stopping problem must be solved.

A computationally simple way of solving the problem is to invoke a high-contact condition at the bankruptcy boundary. The high contact for equity at V_B is equivalent to equity value being continuously differentiable across the bankruptcy boundary and can be written as follows (E^- and E^+ are equity values in bankruptcy and pre-bankruptcy periods, respectively):

$$\partial_V E^- \Big|_{V=V_B} = \partial_{V_B} E^+(V_B) \quad (20)$$

During bankruptcy the debtor solves another similar stopping problem. This time, however, she must weigh the possibility of upside potential with $V=V_R$ against the probability of getting nothing, if the value, V , goes down to θV_B . The high contact in this case is given by

$$\partial_V E^+ \Big|_{V=V_R} = \partial_{V_R} E^+(V_R) \quad (21)$$

Equations (20) and (21) are highly non-linear functions of V_B and χ (defined in Theorem 2). There is no closed form solution for this system. Thus, we solve numerically using Powell's hybrid algorithm, which uses a finite-difference approximation of the Jacobian, to find V_B and χ (and $V_R=\theta\chi V_B$).

The apparent problem with the above formulation is that the high-contact conditions represent only necessary (but not sufficient) conditions for the equity value maximization. High contact is necessary and sufficient only in the case of linear reward and the state variable following a geometric Brownian motion. To make sure that equity value indeed achieves a maximum at $V=V_B$ and at $V=V_R$, one needs to verify the second order conditions for the

stopping problems.³⁷ This requires the explicit expression for the differential operators governing the equity processes in both periods. The expression for the operator in the pre-bankruptcy period has no simple form. Thus, to simplify calculations, we check the optimality in this first period numerically for each case considered. However, in the bankruptcy period the operator has a simple and intuitive form.

$$L_E E^+ = 0 \quad (22)$$

where

$$L_E = .5\omega^2 V^2 \partial_{VV} + (r - b)V \partial_V - r$$

Let $D = \{V: V < V_R\}$ denote the continuation region in which it is not optimal to stop (by proposing an optimal reorganization plan). The debtor's functional (18) is maximized, if and only if a pair (D, E^+) satisfy the following conditions (Oksendal (1998), p. 215)

$$\begin{aligned} L_E E^+ &= 0 && \text{on } D \\ \partial_V E^+_{V=V_R} &= \partial_{V_R} E^+(V_R) && \text{on } \partial D \\ L_E E^+(V) &\leq 0 && \text{outside the closure } \overline{D} \end{aligned} \quad (23)$$

Given that $E^+(V_R) = V_R - K - (1 - \alpha)V_L \left(\frac{V_R}{\theta V_B} \right)^{\gamma_2^+}$, condition (23) becomes

$$bV \geq rK \quad (24)$$

The last condition has an intuitive interpretation. A sufficient condition for the maximum is for the savings from stopping, bV , to be higher than the benefit from continuing and saving interest that would have accrued on a one time fee, K . This gives a natural restriction on the stopping value V_R

$$V_R \geq rK/b \quad (25)$$

I verify this condition for every case to make sure that the maximum is actually achieved.

³⁷ Note, that unlike in previous papers (e.g., Leland (1994), Leland (1998), Leland and Toft (1996)), here the optimality is not based on the equity value being equal to zero at $V=V_B$. Shareholders are able to maintain their value at some positive level even in default.

MODEL APPLICATIONS

In this section, we apply the model developed in the previous section to learn about the magnitude and behavior of credit spreads and average times spent in Chapter 11. In the first part, we calibrate both the model of Leland and Toft (1996) and our model using a method similar to that of Huang and Huang (2002). The models are forced to match certain historical data on default experience. Parameters that make the model consistent with the data are then used to produce the credit spreads. The credit spreads predicted by both models are then compared to the observed yield spreads on bonds of different ratings. While it is intuitive that our extensions to Leland and Toft (1996) will result in higher credit spreads, the scale of these spreads is not obvious. Also, given the highly non-linear nature of the model, its comparative statics are interesting. In the second and the third parts, we discuss empirical implications regarding the average duration of Chapter 11 as a function of the model parameters.

3.1 Calibration of the Model

It has been shown by a number of researchers that the existing credit risk models fail to account for the most part of the observed yield spreads when confronted with data. Our main purpose in the following exercise is to see whether incorporating strategic bargaining of the claimholders in bankruptcy explains a larger fraction of the observed yield spreads. For this purpose, we use a calibration approach proposed recently by Huang and Huang (2002). In this approach, the model is forced to match the data on historical default experience by varying the parameters of the model. The resulting parameter estimates are then used as input in the model to calculate the credit spreads. We compare the credit spreads generated this way for 10-year bonds of different credit ratings with those generated by the LT model.

The values in our model are functions of a 14-dimensional parameter vector, $\Theta = \{V_0, P, C, T, \delta, r, \sigma, b, \omega, \alpha, \theta, K, \tau, \Lambda\}$. Their definitions are listed in Table 1. All calculations are done for $P=100$. The coupon, the payout rate, and the risk-free rate are taken at their historical averages. Specifically, the coupon rate is assumed to be 8%, the payout rate is 7%, and the risk-free rate is 7%. I use τ of 35% - close to the historical average of the Federal tax rate.

The model is then forced to match the following default data taken from Huang and Huang (2002) ³⁸ as well as data on average duration of Chapter 11 bankruptcy and survival probability (likelihood of reorganization) as reported in recent academic studies:

1. leverage for different credit ratings,
2. default rate for bonds of different ratings over a 10-year period,
3. recovery rate in default (defined by Moody's as the allocation to the bondholders at default as a fraction of the face value), and
4. the average time the firms spend in Chapter 11 (also reported by Moody's),
5. Equity premia for different credit ratings (see Huang and Huang (2002)),
6. Liquidation frequency in Chapter 11 ³⁹.

The data for the first five constraints is given in the following table. Leland and Toft (1996) model is calibrated to match the first four columns, whereas my model is calibrated to match all five. This is because our model also gives a closed-form expression for the average time firms spend in bankruptcy.

Target Parameters Used in Calibration

Leverage Ratio (%)	Equity Premium (%)	Cum. Default Prob. (%)	Recovery as % of Par	Average Time in Bankruptcy, years
13.1	5.38	.77	50.0	1.7
21.2	5.60	.99	50.0	1.7
32.0	5.99	1.55	50.0	1.7
43.3	6.55	4.39	50.0	1.7
53.5	7.30	20.63	50.0	1.7
65.7	8.76	43.91	50.0	1.7

³⁸ The data are originally obtained from Moody's Investor's Service and Standard and Poor's Credit Market Services and from several academic publications (see Huang and Huang (2002) for detailed discussions).

³⁹ Empirical studies' estimates of the likelihood of reorganization in Chapter 11 vary quite a lot. Weiss (1990) and Gilson, John and Lang (1990) find that only about 5% of the firms liquidate under Chapter 7 after filing for Chapter 11. Morse and Shaw (1988) find this number to be between 15% and 25%.

Note that in this calibration approach, unlike in the Huang and Huang method, we use the exact expressions for the default boundaries for finite-maturity debt. As stated in the beginning of the chapter, the goal is to learn how much more of the yield spread can be explained by incorporating the bankruptcy process into a structural model. To pursue this goal, we take as the objective function the absolute difference between the model-predicted credit spread and the observed yield spread. The objective is minimized subject to the six constraints listed above. This may be achieved by varying parameters V_0 , σ , b , T , ω , α , θ , K , and Λ .

Numerical results suggest that the values of constraints and the objective are very sensitive to parameters V_0 , σ , b , T and θ and less sensitive to parameters ω , α , K and Λ . In other words, the objective is relatively flatter with respect to ω , α , K and Λ . This allows us to simplify the optimization procedure as follows.

There are six constraints, the first three of those must be solved jointly with computation of default and reorganization triggers, V_B and V_R , in the risk neutral measure. The remaining three constraints are cast in the actual measure and, thus, have to contain the transition from risk neutral to actual measure.

The equity premia for different credit ratings are used to calculate the asset risk premia in the pre-bankruptcy period. Knowledge of the asset risk premia allows us to go from an equivalent martingale measure to the actual measure in the calculation of default probabilities. The implied asset risk premium estimate necessary to compute the default probability is obtained from historical equity premium using the following expression that is a consequence of the fact that equity can be viewed as an option on the underlying asset value in the model:

$$\lambda_V = \lambda_E \frac{E}{V} \frac{\partial V}{\partial E},$$

where λ_V and λ_E are asset and equity risk premia, respectively. The only effect of constraint 5 is to use $r + \lambda_V$ instead of just r in the drift μ for the purpose of calculating default probability as in formula A.1 in the Appendix. Thus, constraint 5 can just be embedded into constraint 2.

The relationship between the equity and asset risk premia just discussed may be appropriate only in the pre-bankruptcy period where it is possible to hedge an exposure in equity risk. However, even if we had data on equity premia for bankrupt firms, in the bankruptcy period it may be difficult or impossible to construct the hedge. For the purpose of adjusting to the actual

measure in Chapter 11, we assume that the asset risk premium is a constant, Λ . Constraint 6 just imposes a lower limit on the probability of reorganization. Given quite dispersed empirical results regarding this probability as mentioned in footnote 39, we constrain it to be at least 50%. The probability of reorganization is much more sensitive to Λ than any other quantity. This allows us to choose Λ essentially independently from the optimization procedure. Numerical results show that $\Lambda=9\%$ is consistent with constraint 6 for all credit ratings. The transition from risk neutral to actual measure is achieved by adding Λ to risk free rate in the formulas for average time in bankruptcy (as discussed in the next section) and the probability of reorganization given in A.14 in the Appendix.

At this point we essentially have to minimize the objective subject to only 4 active constraints. This is done by varying parameters V_0 , σ , b , T and θ . Parameters ω , α , K are chosen to be the same for all six cases of different credit ratings. The values used for these parameters, $\omega=21\%$, $\alpha=14\%$ of the asset value and $K=18\%$ of the face value of debt, were chosen to produce the highest credit spread given that an interior solution has to exist for all credit ratings. The value of θ ranged from .76 for all investment grade issues to .8861 for B-rated debt (Table 2, Panel B).

Table 2, Panel A reports the results of the calibration applied to the Leland and Toft model. This model explains only about 15% of the yield spread on a AAA debt. The LT model explains most of the yield spread only for a speculative grade B debt. We also calibrate our model and report the results in Panel B. Clearly, the explanatory power is significantly better. The model accounts for about a quarter to a third of the observed yield spreads of investment grade Aaa and Aa bonds, half of the observed spread for A-rated bonds and most or the entire spread for Baa, Ba, and B-rated debt.

The reason for this result is related to the properties of the complex option that the debtor possesses in the pre-bankruptcy phase. The option to choose the default point is controlled by the debtor. This is somewhat equivalent to a barrier option with a certain allocation to the holder of the option in the event of reaching the lower boundary (V_B in this case). In the LT model the debtor is frozen out, whereas in my model the allocation to the debtor is positive and endogenously determined as an outcome of the bargaining game between the debtor and the bondholders in the bankruptcy phase. As a result, the debtor exercises the option sooner in my

model than in the LT model. This leads to a higher equilibrium default trigger, V_B , in our model. The price of an individual debt issue is given in equation (6). The first three terms represent the sum of the value of the riskless t -period bond and the present value of the $C/(rT) - P/T$ to be received today with probability equal to that of default. The values of those three terms are controlled by the calibration procedure. The only difference comes from the last component. Its value can be interpreted as the short position on a hypothetical Arrow-Debrew security that pays \$1 at default only if default occurs before the bond matures. It is a short position due to the fact that as a result of calibration the recovery rate, $D(V_B)/P$, is fixed at 0.5. Thus, two results are in place. First, recall that V_B in my model is higher than that in the LT model. A larger V_B would result in a higher default probability, other things equal. Second, the calibration procedure requiring to match the model to the observed default rate results in a substantially lower implied asset value volatility, which would serve to reduce the default probability, other things equal. Both effects are at work in determination of the value of the Arrow-Debrew security. The former effect, however, dominates the latter. Under these circumstances the value of the Arrow-Debrew security is higher in our model than in LT. Thus, the value of debt is smaller and, hence, the credit spreads are higher.

One objection to the above interpretation is that the time to maturity parameter, T , which is increasing due to the calibration of my model, may be responsible for the credit spreads being lower in the LT model than in our model. In Panel C of Table 2, we present the calibration results for the LT model controlling for the time to maturity. In particular, when calibrating the LT model, we set the time to maturity to be equal to that implied by the calibration of our model in each case of different credit rating.

The results show that matching the maturity as described above reduces the credit spreads produced by the LT model dramatically. This finding is in contrast to the intuition provided by Huang and Huang (2002). They claimed that the credit spreads should be higher for debt of longer maturity. However, their argument understates the endogenous nature of the bankruptcy trigger and its dependence on the maturity parameter. As maturity is increased, the debtor defaults later. This would serve to reduce the probability of default over a given horizon and, thus, would reduce the credit spreads, other things equal. However, one of the constraints calls for matching the default probability to its historical level in the calibration. Enforcing this

constraint results in the reduction of pre-bankruptcy asset volatility, which again serves to reduce the credit spreads. This is the reason for the reduction in the explanatory power in the LT model in this case.

The numerical results show that the higher spreads come with large deviations from APR. As in Eberhart, Moore and Roenfeldt (1989) (EMR later), we define a measure of these violations as follows. The average relative deviation from APR in a recapitalization is

$$\delta = \frac{\min(CD, DCS)}{TD},$$

where CD is the average value of bondholder deficiencies, DCS is the average value of distributions to common shareholders, and TD is the average total value of all distributions. As we go from AAA debt to B-rated debt in Table 2, Panel B, our results show that δ ranges from 42.7% for AAA debt to 54.1% for B-rated debt. EMR examined a sample of 30 large firms and found that δ averaged 7.6% over all 30 cases and 9.9% over the 23 cases where it was positive with the maximum attained at 36%. Considering that we model an environment extremely favorable to the debtor, we conclude that modeling Chapter 11 bargaining would not produce much higher spreads than in the LT model if we match the model to the observed magnitude of APR violations. This is in contrast to conclusions in Morellec and Pascal (2002), who model bargaining between the debtor and the bondholders and find that the credit spreads in their model are consistent with the observed yield spreads.

3.2 Expected Duration of Bankruptcy Period

Our model allows us to determine the expected time the firm spends in bankruptcy as a function of the model parameters. There is a tradeoff at each point in the game as seen from equation (18). On one hand higher V_R leads to higher payoff to the debtor at the reorganization boundary. On the other hand it means a lower probability of reaching that value before triggering liquidation. This tradeoff is one of the most important determinants of the mean time spent in bankruptcy. The following result relates the average time in Chapter 11 to model parameters.

Theorem 3. Let χ and $\xi_I(V)$ be as defined in Theorem 2, and $\mu^+ = \frac{r-b-.5\omega^2}{\omega^2}$. Then the expected time a firm spends in the bankruptcy period until it successfully emerges from it (i.e., V reaches level V_R before V_L) is given by the following expression:

$$E[\tau_R | \tau_R < \tau_L] = \frac{1}{\omega^2 \mu^+} \left\{ \ln \chi \frac{1 + \chi^{-2\mu^+}}{1 - \chi^{-2\mu^+}} + \ln \theta \frac{1 + \theta^{2\mu^+}}{1 - \theta^{2\mu^+}} \right\} \quad (26)$$

As mentioned in the previous section, the mean time in bankruptcy in (26) is given in the risk neutral measure. However, the probability of reorganization used in computation of the mean time has to be expressed in the actual measure. To do this, as pointed out in the previous section, we add a constant asset risk premium $\Lambda=9\%$ to the risk free rate. This ensures that the probability of reorganization is at least 50%. This is one of the calibration constraints imposed for consistency with the empirical evidence as described in footnote 39.

To illustrate comparative statics for the average time in Chapter 11, we use A-rated debt as an example with parameter values fixed at levels obtained in the calibration (see Table 2, Panel B).

Table 3 presents the predicted relationship between the liquidation cost and the mean time the firm spends in bankruptcy. As α increases from 2 percent to 14 percent, the expected time in bankruptcy drops from about 4 to 2 years. Higher α means that the bondholders' outside option is less valuable. In the case of liquidation they will receive a lower allocation, as more of the value is lost to liquidation costs. As mentioned previously, the debtor holds a complex option in the bankruptcy period. Among other things, this option has a strike price equal to the sum of the fixed bankruptcy costs, K , and the allocation to the bondholders in a reorganization plan. Thus, the strike price is a highly nonlinear function of all parameters of the model. In particular, it is a decreasing function of percentage liquidation costs, α , other things equal. Higher liquidation costs make the outside option of the bondholders less valuable. The debtor, thus, anticipates larger wealth transfers from the bondholders and exercises her option to default earlier (i.e., at a higher V_B). In addition, the debtor obtains a reduction in the strike price of her option in Chapter 11. This makes a stay in Chapter 11 a costlier process. This is a direct consequence of the fact that the bondholders will be paid less in a reorganization plan of a firm

with higher liquidation costs (Theorem 1). In this situation the optimal response of higher liquidation cost firms is to wrap up the bankruptcy phase earlier. For example, firms with low levels of tangible assets (high synergies, growth opportunities, specific human capital, etc.) are expected to spend less time in bankruptcy. Furthermore, there is a higher chance that the firm will successfully emerge from Chapter 11.

Similar to the result obtained by LT, for higher σ , the debtor optimally chooses to default later (V_B and V_L go down due to limited liability of debtor). The optimal strategy of the debtor facing higher asset volatility in the pre-bankruptcy phase is to default later (at lower V_B). As a result, the firm enters bankruptcy when it is less solvent. Essentially, now the debtor's option can be viewed as an option on an asset paying a lower dividend, bV_B . It is a standard result that this option will optimally be exercised later (Table 4). As a result, as the pre-bankruptcy volatility increases from 10% to 40%, the mean bankruptcy period duration increases from about 1 to 7 years. Credit spreads increase by a factor of about 10 (from 53 bp to 543 bp) to compensate bondholders for the reduction in their expected payoff in the reorganization. The likelihood of reorganization drops slightly from 85% to 80% that is a direct consequence of longer expected duration of bankruptcy.

Firms issuing debt claims of longer maturity optimally choose to default later. Table 5 shows that firms carrying longer maturity debt will on average spend more time in bankruptcy. The intuition is largely the same as before. An option on an asset paying an income stream will be exercised later if the income payments arrive later in the life of the option. Thus, as we extend the maturity of debt in the capital structure, the debtor optimally chooses to default later (at lower V_B). In consequence, the firm enters bankruptcy less solvent and, according to the argument above, spends more time in Chapter 11.

If for a given V_B the volatility of asset value in bankruptcy, ω , were to increase, that would unambiguously increase the probability of liquidation as the drift of the asset return process is reduced. The companion effect is that the reorganization becomes less likely. In particular, the probability of successfully reorganizing in Chapter 11 drops from 83% to about 51% as the bankruptcy period volatility, ω , increases from 21% to 30% (Table 6). Thus, higher bankruptcy period volatility increases the chances that the debtor will not receive anything. To reduce the impact of this event on her value, in equilibrium the debtor optimally delays the

default point (lowers V_B) and spends longer time in bankruptcy. The last result is again due to the fact that now the debtor's option can be viewed as an option on an asset paying lower dividends. As the bankruptcy period volatility increases from 21% to 30%, the expected duration of Chapter 11 grows from about 2 years to just over 5 years (Table 6). In the exclusivity period environment that we adopted in this model, higher volatility of asset value in bankruptcy is always beneficial to the bondholders as they expect to obtain their outside option sooner. As expected, the credit spreads then should drop with ω . Indeed, they decrease from about 64 basis points to just over 4 basis points as ω increases from 21% to 30% (Table 6).

3.3 Conclusion

This dissertation presents a continuous-time model of debt valuation with the possibility of default and Chapter 11 bankruptcy. We examine the bargaining game between the debtor and bondholders in continuous-time using differential game theory. We provide closed-form solutions for the values of equity, finite-maturity debt, and credit spreads. The time to default, Chapter 11 reorganization duration, and allocations to the debtor and bondholders in Chapter 11 are derived endogenously as the outcome of a bargaining game between the debtor and the bondholders.

We provide the calibration results of my model. In doing so, we vary the parameter vector so that theoretical predictions correspond to the observed historical data on default probabilities, equity premia, initial leverage, recovery rates in default, the average time firms spend in Chapter 11 reorganization, and the probability of successful reorganization. Parameter values obtained in this fashion are used to get theoretical credit spreads.

By comparing our calibration results with those based on Leland and Toft (1996), we show that incorporating strategic bargaining in bankruptcy substantially increases the fraction of observed yield spreads explained by predicted credit risk. This is achieved by giving debtor's extreme bargaining power. More precisely, although our results can account for most of or even the entire yield spread on speculative grade bonds, we show that even in this case of the debtor's high bargaining power our model can explain only about 25% to 50% of average yield spreads on investment grade bonds. This suggests that structural models are very likely missing additional

variables like liquidity and differential taxation of corporate and treasury bonds at the state level. Incorporating these additional features may potentially affect equilibrium credit spreads and, thus, improve the predictive ability of the structural approach.

APPENDIX A

PROOFS OF MAIN RESULTS

A.1 Proof of Lemma

To calculate the integrals in equation (4) we will need the expressions for the following expectations

$$E1_{\tau_B < t} = e^{-2\mu X} N(\eta_1) + N(\eta_2), \quad (\text{A.1})$$

$$E1_{\tau_B < t} e^{-r\tau_B} = e^{(-\mu-\nu)X} N(\lambda_1) + e^{(\nu-\mu)X} N(\lambda_2), \quad (\text{A.2})$$

where $\mu = \frac{r - \delta - .5\sigma^2}{\sigma^2}$, $\nu = \sqrt{\mu^2 + \frac{2r}{\sigma^2}}$, $\eta_{1,2}(X, t) = \frac{\pm \sigma^2 \mu t - X}{\sigma \sqrt{t}}$, $\lambda_{1,2}(X, t) = \frac{\pm \sigma^2 \nu t - X}{\sigma \sqrt{t}}$.

Derivation of expressions (A.1) and (A.2) can be found in Karatzas and Shreve (1991), p. 196-197. Notice now that

$$E \int_0^{t \wedge \tau_B} e^{-rs} ds = \frac{1}{r} (1 - Ee^{-r(t \wedge \tau_B)}) = \frac{1}{r} (1 - E1_{t < \tau_B} e^{-rt} - E1_{t > \tau_B} e^{-r\tau_B}) \text{ and}$$

$$E1_{\tau_B > t} = 1 - E1_{\tau_B < t}.$$

Then equation (6) can be written as

$$d(X, t) = \frac{C}{rT} (1 - e^{-rt} [1 - E1_{t > \tau_B}] - E1_{t > \tau_B} e^{-r\tau_B}) + \frac{D_B}{T} E1_{t > \tau_B} e^{-r\tau_B} + \frac{P}{T} e^{-rt} (1 - E1_{t > \tau_B}),$$

which is equivalent to (8) after obvious algebraic manipulations.

Integration in (7) using the above result gives

$$D(X) = \frac{C}{r} + \left(P - \frac{C}{r} \right) \left(\frac{1 - e^{-rT}}{rT} - I_1(X) \right) + \left[D_B - \frac{C}{r} \right] I_2(X),$$

which is identical to expression (9). The integrals $I_1(X)$ and $I_2(X)$ are obtained as follows.

Applying Fubini's theorem in the second equality in A.3 below, we have:

$$\begin{aligned} I_1(X) &= \frac{1}{T} \int_0^T e^{-rt} E1_{\tau_B < t} dt = \frac{1}{T} E \int_0^T 1_{\tau_B < t} e^{-rt} dt = E \left(1_{\tau_B < T} \int_{\tau_B}^T e^{-rt} dt \right) = \\ &= \frac{1}{rT} [E1_{\tau_B < T} e^{-r\tau_B} - e^{-rT} E1_{\tau_B < T}] \end{aligned} \quad (\text{A.3})$$

Similarly, applying Fubini's theorem and performing integration, we obtain an expression for integral $I_2(X)$:

$$\begin{aligned} I_2(X) &= \frac{1}{T} \int_0^T E \mathbf{1}_{\tau_B < t} e^{-r\tau_B} dt = \frac{1}{T} E \int_0^T \mathbf{1}_{\tau_B < t} e^{-r\tau_B} dt = \frac{1}{T} E \mathbf{1}_{\tau_B < T} \int_{\tau_B}^T e^{-r\tau_B} dt = \\ &= E \mathbf{1}_{\tau_B < T} e^{-r\tau_B} - \frac{1}{T} E \tau_B \mathbf{1}_{\tau_B < T} e^{-r\tau_B} \end{aligned} \quad (\text{A.4})$$

Notice also that the second expectation on the right of the last inequality in A.4 can be computed as follows. Function $f(\tau_B, \alpha) = \mathbf{1}_{\tau_B < T} e^{-\alpha\tau_B}$ on $\mathbb{R}_+ \times \mathbb{R}_{++}$ is continuously differentiable in α for almost all τ_B except maybe on a set of measure zero and absolutely integrable, which allows us to interchange the order of differential and expectation operators. Combined with the monotone convergence theorem this leads to the following expression:

$$E \tau_B \mathbf{1}_{\tau_B < T} e^{-r\tau_B} = - \lim_{\alpha \downarrow r} \frac{\partial}{\partial \alpha} E \mathbf{1}_{\tau_B < T} e^{-\alpha\tau_B}$$

To calculate the limit, one needs to remember that $\nu = \sqrt{\mu^2 + \frac{2\alpha}{\sigma^2}}$ should be considered as a function of $\alpha \in \mathbb{R}_{++}$, rather than r , in expression A.2. Differentiating A.2 with respect to α , taking the limit, and combining the results in A.4, after some algebra we obtain:

$$I_2(X) = \frac{1}{\sigma \nu \sqrt{T}} \left[e^{(-\mu-\nu)X} N(\lambda_1) \lambda_1 + e^{(\nu-\mu)X} N(\lambda_2) \lambda_2 \right].$$

A.2 Proof of Theorem 1

Within a class of non-cooperative Markovian Nash equilibria the solution to a dynamic game can be found using standard methods of optimal control theory. At each asset value, V , if the debtor decides to make an offer and stop the game, the offer must be high enough for the bondholders to accept it. Otherwise, the game will continue to the next round. Given that the bondholders accept the offer, the debtor - in attempts to maximize her value - tries to determine the minimum offer amount acceptable by the bondholders. This theorem determines equilibrium values as functions of V , but it is silent about the level of V at which it is optimal for the debtor to offer a reorganization plan. The latter problem is a special case of optimal control problems – an optimal stopping problem. Its solution is provided in section 3.4.

The bondholders' decision variable is binary, accept or reject the offer. Bondholders' optimization task can be described as follows⁴⁰:

$$D(V) = \max_{U_1 \in \{0,1\}} (1 - U_1)\phi_2(V) + U_1 E^V \left[e^{-rdt} D(V + dV) \right] \quad (\text{A.5})$$

subject to the following boundary conditions

$$D(\theta V_B) = (1 - \alpha)\theta V_B, \quad \lim_{V \rightarrow \infty} D(V) < \infty$$

Equation (A.5) is the Hamilton-Jacobi-Bellman equation for the bondholder's optimization problem, where $\phi_2(V)$ represents the optimal offer by the debtor (the debtor's equilibrium strategy), which is held constant at this point. The second term is today's expectation of the present value of the next round debt. This component is called the continuation value of debt. The expectation is taken with respect to process (15).

The first boundary condition specifies that, if the asset value goes down to θV_B , the firm is forced to liquidate with the bondholders receiving the liquidation value and the debtor receiving nothing. The second condition states that debt value must be bounded from above.

It is important to notice that the introduction of a finite exclusivity period will not change anything in the nature of the game as long as the bondholders *do not* have any cram down power. In this case the outside options of the debtor are not affected even after the exclusivity is lifted. Since the bondholders cannot enforce their plan, the debtor can and will always reject any offer by the bondholders that exceeds the bondholder's continuation value.

The solution to (A.5) is rather trivial and can be summarized as follows. Let D be the continuation value of debt (the value of the expectation in (A.5)). Then

$\phi_1=1$ (the bondholders accept the plan) if $\phi_2(V) \geq D$,

$\phi_1=0$ (the bondholders reject the plan) if $\phi_2(V) \leq D$.

At this point we assume that there exists a value, V , at which the debtor decides to end the game and exit the bankruptcy period. The question of what this value is and whether it exists at all is considered later when we solve the debtor's optimal stopping problem.

⁴⁰ Notice that due to the stationary debt structure in the present model, the value of debt is not a function of time.

The debtor's optimal strategy is to offer at least $D+\varepsilon$ to end the game, $\varepsilon>0$.⁴¹ The debtor's functional is strictly decreasing in $D(V)$. Hence, at each V the debtor will never offer more than the continuation value. Thus, the equilibrium is the following pair of strategies:

$$\phi_1(V)=1, \quad \phi_2(V)=D(V).$$

To find the continuation value of debt, expand the right-hand side of

$$D(V) = E^V \left(e^{-rdt} D(V + dV) \right)$$

into a Taylor series. Using (15) and Ito calculus multiplication rules, I obtain the ODE for the value of debt in the continuation region.

$$.5\omega^2 V^2 \partial_{VV} D + (r-b)V \partial_V D - rD = 0$$

subject to the boundary conditions for A.5.

The solution to this problem:

$$\phi_2(V) = D(V) = (1-\alpha)\theta V_B \left(\frac{V}{\theta V_B} \right)^{\gamma_2^+}, \quad \gamma_2^+ < 0$$

Equity value is then given by $E^+ = V^+ - BC^+(V) - \phi_2(V)$. ■

A.3 Proof of Theorem 2

In the bankruptcy period equity is similar to American down-and-out call option. If the value of the state variable, V , drops by $1-\theta$ percent to V_L (relative to the starting point V_B), the debtor receives nothing, and the option is rendered worthless. However, if V reaches V_R , then equity value is $V_R - K - D^+(V_R)$. Thus,

$$E(V) = E([V_R - K - (1-\alpha)V_L \chi^{\gamma_2^+}] e^{-r\tau_R} 1_{\tau_R < \tau_L}) = [V_R - K - (1-\alpha)V_L \chi^{\gamma_2^+}] \xi_1(V) \quad (A.6)$$

where $\xi_1 = E(e^{-r\tau_R} 1_{\tau_R < \tau_L})$.

To determine ξ_1 , let's look at equation (15). The process

$$X_L - \omega^2 \mu^+ t \sim N(0, \omega^2 t)$$

is an \mathcal{F}_t - Brownian motion, where $\mu^+ = \frac{r-b-.5\omega^2}{\omega^2}$.

⁴¹ To simplify notation, it is assumed that the bondholders accept any plan that offers them at least as much as they would obtain by rejecting it (i.e., they accept even if $\varepsilon=0$).

Now we use the standard result that, if Z_τ is stopped \mathcal{F}_τ - Brownian motion, then $M_\tau = \exp\{\gamma Z_\tau - .5\gamma^2 \tau\}$ is a bounded \mathcal{F}_τ - martingale, and τ is a stopping time of the filtration \mathcal{F}_t , $0 \leq t < \infty$, generated by the process X_L up to time t .

Using Optional Sampling Theorem (see Karatzas and Shreve (1991), p. 19), we get the following equality:

$$e^{\gamma X_L} = E e^{\gamma(X_{\tau_R \wedge \tau_L \wedge t} - \omega^2 \mu^+ (\tau_R \wedge \tau_L \wedge t)) - .5\omega^2 \gamma^2 (\tau_R \wedge \tau_L \wedge t)}$$

Now taking a limit as $t \rightarrow \infty$ and using monotone convergence theorem we get:

$$e^{\gamma X_L} = E 1_{\tau_R < \tau_L} e^{\gamma X_{\tau_R} - (\gamma \omega^2 \mu^+ + .5\omega^2 \gamma^2) \tau_R} + E 1_{\tau_R > \tau_L} e^{-(\gamma \omega^2 \mu^+ + .5\omega^2 \gamma^2) \tau_L}$$

To calculate ξ_1 , notice that equation $\gamma \omega^2 \mu^+ + .5\gamma^2 \omega^2 = r$ has two different solutions,

$$\gamma_{1,2} = -\mu^+ \pm \sqrt{(\mu^+)^2 + \frac{2r}{\omega^2}} \quad (\text{A.7})$$

Hence, we have a system of two equations for ξ .

$$\begin{cases} e^{\gamma_1 X_L} = e^{\gamma_1 X_R} \xi_1 + \xi_2 \\ e^{\gamma_2 X_L} = e^{\gamma_2 X_R} \xi_1 + \xi_2 \end{cases} \quad (\text{A.8})$$

The solution exists and is unique iff $\gamma_1 \neq \gamma_2$ and $X_R \neq 0$. The solution is given by

$$\xi_1 = \frac{e^{\gamma_1 X_L} - e^{\gamma_2 X_L}}{e^{\gamma_1 X_R} - e^{\gamma_2 X_R}}; \quad \xi_2 = \frac{e^{\gamma_1 (X_L - X_R)} - e^{\gamma_2 (X_L - X_R)}}{e^{-\gamma_1 X_R} - e^{-\gamma_2 X_R}}.$$

A.4 Proof of Theorem 3

We are interested in the average time distressed companies, that successfully reach the reorganization point after declaring themselves bankrupt at V_B , spend in the bankruptcy period. Thus, we are dealing with the truncated distribution of first passage times of V from V_B to V_R . The expected discount factor over a period τ_R

$$\xi_1(V_B) = \lim_{V \downarrow V_B} \xi_1(V) = \frac{\theta^{-\gamma_1^+} - \theta^{-\gamma_2^+}}{\chi^{\gamma_1^+} - \chi^{\gamma_2^+}} \quad (\text{A.10})$$

The mean of the truncated distribution is given by

$$E_{V_B}(\tau_R | \tau_R < \tau_L) = \frac{E_{V_B}(\tau_R 1_{\tau_R < \tau_L})}{P^{V_B}(\tau_R < \tau_L)} \quad (\text{A.11})$$

A.4.1 Evaluation of the expectation in the numerator of (A.11)

Let

$$\xi_1(V_B, \alpha) = E_{V_B} e^{-\alpha \tau_R} 1_{\tau_R < \tau_L} = \frac{\theta^{-\gamma_1^+} - \theta^{-\gamma_2^+}}{\chi^{\gamma_1^+} - \chi^{\gamma_2^+}}, \quad (\text{A.12})$$

where both γ_1^+ and γ_2^+ are now the following functions of α (just as in A.7, but α is substituted for r):

$$\gamma_{1,2}^+ = -\mu^+ \pm \sqrt{(\mu^+)^2 + \frac{2\alpha}{\omega^2}}$$

Differentiating A.12 with respect to α and applying monotone convergence theorem we obtain

$$\begin{aligned} E_{V_B} \tau_R 1_{\tau_R < \tau_L} &= -\lim_{\substack{\alpha \downarrow 0 \\ V \downarrow V_B}} \frac{\partial}{\partial \alpha} E_V e^{-\alpha \tau_R} 1_{\tau_R < \tau_L} = \lim_{\substack{\alpha \downarrow 0 \\ V \downarrow V_B}} \frac{\partial}{\partial \alpha} \frac{\theta^{-\gamma_2^+} - \theta^{-\gamma_1^+}}{\chi^{\gamma_1^+} - \chi^{\gamma_2^+}} = \\ &= \frac{\ln \chi [1 + \chi^{-2\mu^+}] [1 - \theta^{2\mu^+}] + \ln \theta [1 - \chi^{-2\mu^+}] [1 + \theta^{2\mu^+}]}{\omega^2 \mu^+ [1 - \chi^{-2\mu^+}]^2} \end{aligned} \quad (\text{A.13})$$

A.4.2 Evaluation of the probability in the denominator of (A.11)

Similarly to derivation of A.13, taking α to zero and making use of the monotone convergence theorem lead to

$$P^{V_B}(\tau_R < \tau_L) = E_{V_B} 1_{\tau_R < \tau_L} = \lim_{\substack{\alpha \downarrow 0 \\ V \downarrow V_B}} E_V e^{-\alpha \tau_R} 1_{\tau_R < \tau_L} = \frac{1 - \theta^{2\mu^+}}{1 - \chi^{-2\mu^+}} \quad (\text{A.14})$$

The result in (26) is obtained by combining the expressions A.13 and A.14 in A.11.

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